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Hwang, Chiun-Lin, Ph.D.

Iowa State University, 1989

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Optimal monetary policy

in an open macroeconomic model

with rational expectation

by

Chiun-Lin Hwang

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1 INTRODUCTION

This study is concerned with the choice of the optimal control rule of money supply for a linear stochastic discrete time model of an open economy where expectations are rational, in the sense that they are consistent with the model, and where the model includes forward-looking variables. We are confined ourselves to the optimal feedback money supply rule which minimizes a social welfare loss function for an economy with rational expectation. The optimal control theory is applied to the formulation of the stabilization policy under the assumption that the economic system is controllable.

The dynamic optimal control method of Murata (1982) is utilized to derive the optimal money supply rule in our stochastic discrete-time macroeconomic model. The features and merits of the method are that (1) the setting of the optimal control rule is directly applicable to the discrete-time linear economic system which are more attractive to economic researchers than continuous time system[1], (2) in addition to the feedback on past observations, the public expectations have been incorporated directly into the formulation of the control rule which is contrasted to the usual feedback control rule, (3) the optimal control rule is formulated under

[1]Levine and Currie (1985, 1987) have presented the optimal and suboptimal feedback rules for linear stochastic continuous time models with rational expectations and are concerned with time-consistent rule.

the assumption that the economic system is convergent to the steady-state equilibrium with constant mean and asymptotic variance of the state variable, so that the condition that the mean and asymptotic variance-covariance of our concerned endogenous variable are constant is added to be a restriction in controlling the dynamic economic system. In other words, the steady-state equilibrium with constant mean and asymptotic variance-covariance of our concerned endogenous variables will be achieved under the derived control rule for a controllable economic system.

'Optimal' is in the sense that the fluctuation of the economic system is minimized over time. In this sense, optimal control theory is powerful and useful technique for analyzing dynamic systems[2]. Like the standard control problem, the loss function is in quadratic form and time horizon is infinite. Government is assumed to formulate a setting of stabilization policy to smooth the fluctuations of the economic system evolving over time. The goal of the stabilization policy of the government is assumed to be in attempt to minimize a weighted sum of squares of deviations of the state variables (our concerned variables) from their targeted values (or time trend level). Money supply is chosen by government as policy instrument for the setting of the rule to control the evolution of the economic system in our model.

The central practical issue separating Keynesian from non-Keynesian economists is the nature of the setting of the optimal policies. Keynesian economists are in favor of feedback rules. They advocate that policy should 'look at everything' incorpo-

[2]The application of dynamic control theory to the economic planning has been popular and widely used since 1970s. However, some economists (Kyland and Prescott (1977), Lucas (1976)) have argued that control theory is not the appropriate tool for dynamic economic planning, and shown that the policy selected is consistent but not optimal due to the dependence of the economic agent's decision on the expected future government policy.

rating feedback from current and past observations on the state of the economy to the future settings of fiscal and monetary instruments. The fine-tuning policy is called for higher taxes and lower growth rate of money supply in the boom, and lower taxes and higher growth rate of money supply in the recession.

However, non-Keynesian economists such as Friedman (1948 and 1959) advocate that the government follows rules instead of discretionary policies. Furthermore, they argue that the government must set money supply growing at a constant rate per year.

The debate of these two schools concerning the optimal policies is long-standing and durable. The adoption of feedback rule to combat business cycle is originally based on the belief that the probability distribution of output will be influenced by the government policy instrument. The government money supply rule will change the probability distribution of output.

However, some economists[3] have demonstrated that, if expectations are formed rationally, the probability distribution of output is independent of the deterministic money supply rule. They furthermore assert that one deterministic money supply rule is as good as any other, insofar as concerns the probability distribution of real output of an economy with Lucas-type supply and rational expectations. This is the 'neutrality' or 'policy ineffectiveness' proposition. In brief, this proposition provides the policy implication that demand management policy will be ineffective in influencing the time path of real output in an economy in which expectations are rational and money illusion is absent.

The main issues we are concerned are (1) to examine the validity of 'policy

[3]See Lucas (1972, 1973, 1976), Sargent (1973, 1976), Sargent and Wallace (1975).

ineffectiveness proposition', (2) to investigate the comparative performance of the alternative money supply rules - feedback versus fixed rule, (3) to determine whether the structure of the economy is crucial to the effectiveness of the setting of the money supply rule in the sense of the insulation of the economy from the random shocks.

More specifically, what we are concerned about the first issue is whether the probability distributions of endogenous variables are affected under the selected policy rule of money supply for an economy with rational expectations. If the mean and/or variance of endogenous variables of our concern have been changed once the control rule of policy instrument has been adopted, it implies that the policy rule is effective, rather than powerless, even when the expectation is rational.

For the second issue, the relative performance of the alternative money supply rules shall be compared and examined according to the criteria of minimum variances of the state variables. The selection of the appropriate policy relies on the degree of the insulation of the fluctuations of the economy originating from a variety of random disturbances.

For the third issue, we shall explore whether the 'optimal' control rule depends on the structure and nature of the economy.

First, the dynamic optimal control method is applied to Turnovsky-Bhandari model with ad hoc equations. The alternative money supply rules under the alternative objective functions with different target variables are derived to examine the effect of the policy control on the time path of the endogenous variables in the economic system.

Secondly, the drawbacks of the macroeconomic model with ad hoc equations are pointed out, in particular, in the aspects of the functional forms of consumption and money demand appearing in T-B model. Theoretically, the function of consumption and money demand must be simultaneously determined from the optimization behavior of the individual. Then, we are in attempt to provide the nature of demand functions for consumption and money based on the insights from two branches of microeconomic foundations of monetary model - the general equilibrium cash-in-advance (CIA) and overlapping generations model (OLG).

These new developments in the microfoundations of monetary theory arise from the dubious character of conventional assumptions in macroeconomic models[4].

The supporters of the new microeconomic foundations of monetary model point out that the standard portfolio specification (money demand function) included in all the macroeconomic literature cannot be rationalized by appeal to some risk-aversion model and/or some inventory or transaction cost model.

In general, the function of money as a medium of exchange is emphasized more in the CIA model, where the cash-in-advance constraint is used to provide a determinant demand for the government non-interest-bearing money. A central feature of the CIA model is the exogenously imposed requirement that goods be purchased with currency accumulated in advance. However, the role of money as a store of value is more important in the OLG model, where wealth is carried from period to period in the form of money.

Here we are attempting to provide a reasonable consumption and money demand functions, which will be based on the microeconomic foundations of the CIA and OLG models. Two alternative models are developed by replacing demand

[4]Kareken and Wallace (1980) demonstrate rigorously that "macroeconomic models, small-scale and large scale, classical and Keynesian, are inconsistent; they yield contradictory implications".

for money and consumption in the T-B model with the new ones. Finally, another setting of optimal money supply is formulated under the assumption that the monetary authority has information superior to that of the public. The optimal superior-information money rule is derived for the revised CIA and OLG models. It is shown that the optimal money supply rule depends on the structure of the economy.

2 TURNOVSKY-BHANDARI MODEL WITH AD HOC EQUATIONS

Turnovsky and Bhandari (1982) developed a general equilibrium macroeconomic model of a small open economy with imperfect capital mobility under flexible exchange regime. The model is built in the ad hoc equations. Basically, the model having the usual open macroeconomic framework which is widely used to analyze the exchange rate determination by most international economists, follows the conventional macroeconomic analytical framework with rational expectations[1]. The model is extended by incorporating the real sector from Driskill-McCafferty (1980) analysis in which the degree of capital mobility has been taken into account. The characteristic of the model is its consideration of the degree of mobility in the exchange rate determination of capital account, within a framework incorporating both financial and real sectors of the economy. More specifically, the introduction of the imperfect capital mobility into the capital account of balance of payments is contrasted with other open macroeconomic model where capital is assumed to be perfectly mobile internationally. This assumption of perfect capital mobility gives the current account balance, together with asset market, an important role in

[1]Dornbusch (1976), Barro (1978) and Bilson (1978).

determining the short-run exchange rate[2].

The open economy in the model is assumed to be small enough to regard the world price and interest rate as exogenously determined. The public and government form their expectations rationally and share with all the available information. The individuals in the economy engage in the transactions in goods market and a world-wide capital market where domestic and foreign government one-period bonds are traded. Foreign bonds are regarded by domestic agents as being imperfect substitutes of domestic bonds due to a certain sorts of risks and uncertainty involved[3].

Their paper primarily focuses on the two issues: (1) the short-run impacts of various structural disturbances of domestic and foreign origins on the domestic economy, (2) the effects of the degree of capital mobility on the variations of all the endogenous variables.

2.1 The Structure of Turnovsky-Bhandari Model

The model includes four main building blocks: a specification of aggregate supply, of money market equilibrium, of purchasing power parity, and of the balance of payments. The model is built in the following ad hoc equations.

[2]Under the assumption of perfect mobility, the exchange rate is essentially determined in the money market.

[3]As mentioned in Turnovsky-Bhandaris' paper, two factors may create impediments to capital mobility. The first is exchange risk which is associated with the domestic and foreign bonds denominated in different currencies and the uncertainty of exchange rate at maturity. The second is default risk arising from the fact that investors may perceive that foreign bonds are subject to the risk of default more than domestic bonds.

1. Domestic aggregate supply

$$y_t = \bar{s} + s(p_t - E_{t-1}p_t) + u_t, \quad s > 0 \tag{2.1}$$

where $u_t \sim iid(0, \sigma_u^2)$

2. Domestic money market equilibrium condition

$$m_t - p_t = -\beta_1 r_t + \beta_2 y_t \tag{2.2}$$

3. Purchasing power parity

$$p_t = p_t^* + e_t \tag{2.3}$$

where $p_t^* \sim iid(0, \sigma_{p^*}^2)$

4. Net position in foreign bonds, expressed in real terms

$$\frac{B_t}{P_t^*} = \eta(E_t e_{t+1} - e_t + r_t^* - r_t), \quad \eta > 0 \tag{2.4}$$

where $r_t^* \sim iid(0, \sigma_{r^*}^2)$

5. Real trade balance, ignoring domestic investment and government expenditure

$$T_t = y_t - c_t \tag{2.5}$$

6. Balance of payments equilibrium

$$\frac{\Delta B_t}{P_t^*} = T_t, \quad \text{where } \Delta B_t = B_t - B_{t-1} \tag{2.6}$$

7. Domestic aggregate demand

$$c_t = c_1 y_t - c_2[r_t - (E_t p_{t+1} - p_t)] + w_t, \quad 0 < c_1 < 1, \quad c_2 > 0 \tag{2.7}$$

where $w_t \sim iid(0, \sigma_w^2)$

8. Domestic nominal money supply is governed by a stationary process

$$m_t = \overline{m} + v_t \tag{2.8}$$

where

y_t	= real domestic output
p_t	= logarithm of the domestic price level
m_t	= logarithm of the domestic nominal money supply
e_t	= logarithm of the exchange rate
p_t^*	= logarithm of foreign price level
P_t^*	= the foreign-currency price of the goods
T_t	= real domestic trade balance
c_t	= real domestic consumption demand
r_t	= domestic nominal interest rate
r_t^*	= foreign nominal interest rate
B_t	= net position of the domestic agents in foreign bonds measured in the foreign currency
u_t	= random aggregate supply disturbance
w_t	= random aggregate demand disturbance
v_t	= random domestic monetary disturbance
$E_t x_{t+1}$	= the expectation of x_{t+1} conditional on the information set Ω_t

There are seven endogenous variables, y_t, p_t, e_t, r_t, c_t, B_t, and T_t, one domestic exogenous variable, m_t, and five random disturbances, p_t^*, r_t^*, u_t, v_t, and w_t, of which the former two are foreign origins and the latter three are domestic origins.

The country is assumed to produce only a single final homogeneous commodity, or a composite commodity. Equation (2.1) is a simple Phillips curve or Lucas-type aggregate supply equation. The supply function embodies the natural rate hypothesis that only unanticipated increases in the price level are posited to boost aggregate supply. u_t is a random unsystematic supply innovation and is assumed to be identically and independently distributed with zero mean and a finite variance, σ_u^2. For convenience, the level of 'normal' aggregate supply ($\bar{s}$) is set equal to zero.

Equation (2.2) is the standard money market equilibrium equation which describes demand for real money balance as a function of interest rate and real income. Demand for real money balance is inversely to interest rate and directly related to real income. Equation (2.3) is simply purchasing power parity (PPP). The PPP holds under the assumption that the goods is traded goods and no transportation cost incurred. It implies that there is no exploitable profit opportunity for the goods arbitrage internationally. The foreign price, p_t^*, is assumed to be a random variable and identically and independently distributed with zero mean and a finite variance, $\sigma_{p^*}^2$.

B_t/P_t^* in equation (2.4) is the net position of the domestic economy in foreign bonds, expressed in real terms. It is defined as the difference between domestic demand for foreign bonds and foreign demand for domestic bonds. The coefficient η is the measure of the sensitivity of domestic and foreign wealth holder's portfolio composition to the uncovered interest differential in favor of the foreign bond[4].

[4]The uncovered interest parity in an uncertainty world is $r = r^* + E_t e_{t+1} - e_t$. However, for a certainty world in which foreign and domestic bonds are perfect substitutes, the arbitrage conditions for both foreign and domestic assets are the uncovered interest parity, $r_t = r_t^* + e_{t+1} - e_t$.

The size of η reflects the degree of capital mobility and is negatively related to the relative risk averse[5]. For a small country, the supply of foreign bonds is assumed to be perfectly elastic at the existing interest rate yield. The specification of net demand for foreign bond in equation (2.4) can be derived from portfolio optimization of the individual investor behavior under these assumptions that (1) domestic and foreign bonds are perfect substitutes on a covered basis and (2) individual investor is risk adverse. The random foreign interest rate is identically and independently distributed with zero mean and a finite variance $\sigma^2_{r^*}$.

Equation (2.5) is simply a gross national product identity, ignoring domestic investment and government expenditure. Equation (2.6) is balance of payment equilibrium equation with flexible exchange regime and net international interest payments small enough to be ignored. It follows that trade surplus equates net demand for foreign bond. Under flexible exchange regime, trade account balance is exactly equal to the opposite of capital account balance since official settlement balance is always zero[6].

In the model, net capital outflow at time t is defined as the amount of net purchase of foreign bonds by domestic investors at time t over the amount at time $t-1$. It reflects the changes in net position of the domestic economy in foreign bonds. Equation (2.7) is domestic aggregate demand with only one component of domestic aggregate consumption since domestic investment and government expenditure are ignored. Domestic consumption is a function of current real income and real

[5]The infinite η ($\eta \rightarrow \infty$) is associated with the risk neutrality.

[6]In the balance of payments account, net export of goods and services is regarded as a debit item in current account. Net purchase of foreign asset (capital outflow) is recorded as a debit item in capital account.

interest rate, $r_t - (E_t p_{t+1} - p_t)$[7]. Domestic consumption is positively related to current real output and inversely to real interest rate[8]. The random term w_t characterizes the unanticipated changes in consumption expenditures and is identically and independently distributed with zero mean and a finite variance.

Equation (2.8) indicates that the money supply follows a stationary process with a systematic component of money supply ($\overline{m}$). For convenience, $\overline{m}$ is set to be zero. The random term v_t is identically and independently distributed with zero mean and finite variance(σ_v^2). v_t is regarded as a positive money supply innovation or a negative money demand disturbance.

2.2 Rational Expectations Solutions

The rational expectation solutions of endogenous variables are summarized as follows:

For notational convenience, define

$$z_t \equiv e_t + r_t - r_t^* \tag{2.9}$$

1. One period rational expectation forecasts of price level and exchange rate formed at time t are

$$E_t p_{t+1} = E_t e_{t+1} = \frac{\lambda_1}{\lambda_2} z_t, \quad \forall t \tag{2.10}$$

[7]Fisher's definition of real interest rate is the nominal interest rate, r_t, minus the expected inflation rate, $(E_t p_{t+1} - p_t)$, where $E_t p_{t+1}$ is the expected price at time $t+1$ formed at time t.

[8]The real foreign interest income in terms of domestic currency, $(r^* - \Delta P_t^*/P_t^*)B_t/P_t^*$, where $\Delta P_t^*/P_t^*$ is foreign inflation rate, is assumed to small enough to be neglected. Therefore, it is not treated as a determinant factor in consumption equation.

where $\lambda_1 = \eta/(\eta + c_2) < 1$, $\lambda_2 = 1 + 1/\beta_1 > 1$, λ_1 and λ_2 are the roots of the characteristic equation

$$(\eta + c_2)\lambda^2 - [\eta + (\eta + c_2)(1 + \frac{1}{\beta_1})]\lambda + \eta(1 + \frac{1}{\beta_1}) = 0 \qquad (2.11)$$

2. The solution of all the endogenous variables are

$$\begin{aligned}
(a) z_t &= \lambda_1 z_{t-1} + \xi_{1t} \\
(b) y_t &= \xi_{2t} \\
(c) e_t &= \frac{\lambda_1}{\lambda_2} z_{t-1} + \xi_{3t} \\
(d) p_t &= p_t^* + \frac{\lambda_1}{\lambda_2} z_{t-1} + \xi_{3t} \\
(e) r_t &= \frac{\lambda_1}{1 + \beta_1} z_{t-1} + \xi_{1t} - \xi_{3t} + r_t^* \\
(f) T_t &= (1 - c_1)\xi_{2t} + c_2(1 - \frac{\lambda_1}{\lambda_2})(\lambda_1 z_{t-1} + \xi_{1t}) + c_2(p_t^* + r_t^*) - \theta_t
\end{aligned} \qquad (2.12)$$

where ξ_{it} is the composite random variable expressed as linear functions of the structural random variables, $\xi_{it} \sim iid(0, \sigma^2_{\xi i})$.

$$\begin{aligned}
D &\equiv [s\beta_2 + s(1 - c_1)\beta_1\phi + (1 + \beta_1)] > 0 \\
\phi &\equiv \frac{1 + \beta_1}{\eta + c_2(1 + \beta_1)} \\
\xi_{1t} &= \frac{\phi}{D}\{-(1 - c_1)(1 + \beta_1)u_t - s(1 - c_1)v_t + [s\beta_2 + (1 - \beta_1)]w_t \\
&\quad -[c_2(1 + \beta_1 + s\beta_2)] + s(1 - c_1)\beta_1](p_t^* + r_t^*)\} \\
\xi_{2t} &= \frac{1}{D}\{(1 + \beta_1)u_t + sv_t + s\beta_1\phi w_t + s\beta_1(1 - c_2\phi)(p_t^* + r_t^*)\} \\
\xi_{3t} &= \frac{1}{D}\{-[\beta_2 + \beta_1\phi(1 - c_1)]u_t + v_t + \beta_1\phi w_t \\
&\quad -[c_2\beta_1\phi + s(1 - c_1)\beta_1\phi + (1 + s\beta_2)]p_t^* - \beta_1(1 - \phi c_2)r_t^*\}
\end{aligned}$$

The feature of the solutions is that domestic real income fluctuates randomly around its steady value (or trend value) of zero, while other endogenous variables such as the exchange rate, price level, and interest rate follow convergent time-dependent processes.

2.3 Interpretations of the Rationale of Ad Hoc Equations

The model is constructed in the conventional macroeconomic setup with a set of so-called ad hoc equations. Such an ad hoc equation setup is vulnerable if no behavioral rationale is embodied in these equations. The macroeconomic framework in ad hoc fashion has endured severe criticism for a long time from the economists who have a strong taste for microeconomic analysis. We shall succinctly describe the rationale of these equations which have been provided by some economists.

2.3.1 Aggregate supply

The aggregate supply function of equation (2.1) embodies the "natural rate" or "accelerationist" notion that output supply is affected by the unanticipated price level. Only the unanticipated component of inflation is stimulative in the supply of production. The idea of "natural rate" hypothesis has been put forth by Friedman (1968), Phelps (1967), and others[9].

According to Friedman's discussion, it is postulated that demand for labor depends on the real wage while the supply of labor depends on the expected real wage. This argument is based on the idea that individual employer has full knowledge of

[9]McCallum (1980) surveyed a couple of well-known and widely accepted rationales of such aggregate supply.

input and output prices but individual employee has imperfect knowledge of output price. The equations of demand for labor and supply of labor are expressed as

$$(a) \quad n_t = a_0 + a_1(w_t - p_t) + u_{1t}, \quad a_1 < 0$$
$$(b) \quad n_t = a_2 + a_3(w_t - E_{t-1}p_t) + u_{2t}, \quad a_3 > 0 \tag{2.13}$$

where n_t, w_t, and p_t are logarithms of employment, the nominal wage and price level, respectively. u_{1t} and u_{2t} are random disturbance terms which are identically and independently distributed with zero mean and finite variance. The term $(w_t - E_{t-1}p_t)$ is the expected real wage. By the labor market equilibrium condition that demand for labor equates labor of supply, it is obtained that

$$n_t = a_4 + a_5(p_t - E_{t-1}p_t) + u_{3t}, \quad a_5 > 0 \tag{2.14}$$

where

$$\begin{aligned} a_4 &= a_0 + \frac{a_1(a_2 - a_0)}{(a_1 - a_3)} \\ a_5 &= \frac{a_1 a_3}{a_1 - a_3} \\ u_{3t} &= \frac{-a_3}{a_1 - a_3}u_{1t} + \frac{a_1}{a_1 - a_3}u_{2t} \end{aligned}$$

Thus, by a production function relating output to employment, the equation (2.14) can finally become the relation like

$$y_t = \bar{s} + s(p_t - E_{t-1}p_t) + u_t \tag{2.15}$$

It is noted here that in general the lagged output is included in Lucas-type aggregate supply function[10].

[10]Sargent (1979) provides an adjustment-cost rationale for the appearance of y_{t-1} in the aggregate supply function. But, it must be noticed here that the existence of adjustment cost is not necessary for the appearance of y_{t-1}.

Lucas (1973) provides an alternative rationale based on the idea that individual supplier cannot accurately distinguish the movement between economy-wide and relative price in his own locality. The individual supplier's decision depends only on relative price. Lucas supposes that output suppliers are located in a large number of separated competitive markets. Demand for goods is distributed unevenly over market, leading to economy-wide and relative price movements.

2.3.2 Net demand for foreign bond

We shall proceed to derive net demand for foreign bond from the portfolio optimization behavior of individual investor[11]. Assume that the domestic individual investor is allowed to hold interest-bearing assets of domestic and foreign bonds. There is no institutional obstacle or restriction on the portfolios of bonds across countries. But the imperfect capital mobility arises from the fact that some sorts of uncertainty and risk have been taken into account in the individual investor's portfolio behavior.

The representative domestic investor holds domestic and foreign one-period bonds. For simplification, we consider the net foreign bonds denominated in the foreign currency. B_t is the quantity of net foreign bonds held by domestic investor at the end of period t. The rate of return net of opportunity cost per unit B_t is $(e_{t+1} - e_t + r_t^* - r_t)$. Hence the profit of net foreign bond holdings is

$$\Pi_{t+1} = B_t(e_{t+1} - e_t + r_t^* - r_t) \tag{2.16}$$

The optimization problem of the domestic representative investor's portfolio

[11]Black (1985) has presented the equation of the net demand for asset emerged from individual investor's portfolio behavior.

behavior is to choose B_t so as to maximize the expected utility of profit,

$$E(U_t) = E_t\Pi_{t+1} - \frac{R}{2}V_t\Pi_{t+1} \tag{2.17}$$

where

R is the measure of risk aversion

$E_t\Pi_{t+1}$ is conditional expectation value of profit, given information available at time t

$V_t\Pi_{t+1}$ is conditional variance of profit, given information available at time t.

Since

$$\begin{aligned} E_t\Pi_{t+1} &= B_t(E_te_{t+1} - e_t + r_t^* - r_t) \\ V_t\Pi_{t+1} &= E_t[B_t(e_{t+1} - E_te_{t+1})]^2 = B_t^2\sigma_{e,t+1}^2 \end{aligned} \tag{2.18}$$

where $\sigma_{e,t+1}^2$ is the conditional variance of the next period's exchange rate, thus the objective function of equation (2.17) becomes

$$E(U) = B_t(E_te_{t+1} - e_t + r_t^* - r_t) - \frac{R}{2}B_t^2\sigma_{e,1}^2 \tag{2.19}$$

where $\sigma_{e,1}^2$ is one-period variability of the exchange rate.

Taking the first derivative with respect to B_t, the necessary condition for this expected utility maximization problem is

$$\frac{dE(U)}{dB_t} = (E_te_{t+1} - e_t + r_t^* - r_t) - RB_t\sigma_{e,1}^2 = 0 \tag{2.20}$$

Solving equation (2.20) for B_t, the optimal net foreign bond holdings is easily obtained,

$$B_t = \frac{1}{R\sigma_{e,1}^2}(E_te_{t+1} - e_t + r_t^* - r_t)$$

$$= \eta(E_t e_{t+1} - e_t + r_t^* - r_t)$$

where $\eta \equiv 1/R\sigma^2_{e,1}$ is the representative investor's risk-bearing behavioral parameter, which is dependent on the degree of risk aversion R and one-period variability of the exchange rate, $\sigma^2_{e,1}$. η parameterizes the degree of capital mobility. The larger value η means the more capital mobility. The value of η is inversely related to R and $\eta^2_{e,1}$. If domestic investor is more risk aversive towards risk (the larger R), then capital is less mobile across countries (small η). Likewise, if the variation of exchange rate of one-period ahead is larger, the capital is less mobile across countries.

3 OPTIMAL MONETARY POLICY OF T-B MODEL

We have reviewed the T-B model where money supply follows a stationary exogenous process. Now, our attention is confined to the formulation of the stabilization policy under rational expectations. The emphasis is put on the choice of the optimal money supply rule by virtue of dynamic optimal control method. An optimal control method, in particular, for linear discrete-time economic systems presented by Murata (1982) will be applied to T-B model and then derive the optimal feedback money supply rule.

The main purpose of this chapter is, taking T-B model as an example, to examine the relative performance of alternative money supply policies under the alternative government objective functions. Under the fixed exchange rate system, money supply is endogenous which is partly determined by trade balance. The authority would intervene in the exchange market to avoid exchange appreciation by buying foreign exchange if the country has trade surplus. On the contrary, the authority would intervene in the exchange market to offset exchange depreciation by selling foreign exchange if the country has trade deficit. Therefore, foreign exchange reserves, one component of money supply, would increase or decrease leading to money supply changes. But under the fully flexible exchange regime, it is assumed that there is no government intervention in exchange market so that foreign exchange reserve is fixed. Under the flexible exchange rate, money supply is essentially exogenous which can be controlled by the monetary authority. Money supply serves as a superior monetary policy instrument to interest rate under the flexible exchange regime[1].

The government authority is assumed to adopt the feedback money supply rule as a countercyclical policy to combat business cycle and/or to eliminate the serial correlation of endogenous variables. The alternative optimal feedback money supply rules for this small open economy with rational expectation are derived by utilizing the dynamic optimal control method in the infinite time-horizon problem. In the setting of dynamic optimal control model, the key assumption is the steady-state equilibrium of all the state variables. This assumption is required to be added as the constraints in solving the optimal control instrument of money supply. 'Optimal' is in the sense that the fluctuations of the target variables from their trend values (or targeted values) are minimized.

In this chapter, we proceed to derive the optimal feedback money supply rule in an economy in which the government takes into account the public expectations of price and exchange rate in the setting of feedback monetary policy. We are in attempt to examine the variability of the "policy ineffective proposition".

The final forms of all the endogenous variables under the optimal control of money supply are derived with a view to examining the sources of variations of all the endogenous variables and the impacts of all the random disturbances of domestic and foreign origins on the variations of the endogenous variables.

[1]Poole (1970) discussed the selection of instrument between interest rate and money supply policies in a closed economy.

The alternative feedback money supply rule will be subsequently derived with the alternative stabilization criteria. In addition, the fixed money supply rule is derived as well. The relative effectiveness of the alternative monetary policies are justified in accord with the criterion of the minimum variance.

3.1 Three Target Variable Case

First, suppose that these endogenous variables of real output, price level and exchange rate are the monetary policy maker's main concern. Therefore, in the setting or formulation of the dynamic optimal control, the real income (y_t), price level (p_t) and exchange rate (e_t) are treated as state variables and money supply is chosen as control variable.

Taking the first difference of equation (2.4) and combining with equations (2.5) and (2.6), we obtain

$$\begin{aligned} y_t - c_t &= \eta[(E_t e_{t+1} - E_{t-1} e_t) - (e_t - e_{t-1}) + (r_t^* - r_{t-1}^*) \\ &\quad -(r_t - r_{t-1})] \end{aligned} \tag{3.1}$$

The complete model given by equations (2.1)-(2.7) can be reduced to the four equations:

$$(a) \quad y_t = s(p_t - E_{t-1} p_t) + u_t$$

$$(b) \quad m_t - p_t = -\beta_1 r_t + \beta_2 y_t, \quad \beta_1, \beta_2 > 0$$

$$(c) \quad p_t = p_t^* + e_t$$

$$\begin{aligned} (d) \quad &\eta[(E_t e_{t+1} - E_{t-1} e_t) - (e_t - e_{t-1}) + (r_t^* - r_{t-1}^*) - (r_t - r_{t-1})] \\ &= (1 - c_1) y_t + c_2[r_t - (E_t P_{t+1} - p_t)] - w_t \end{aligned} \tag{3.2}$$

The model can be reduced as follows:

$$\begin{pmatrix} 1 & -s & 0 \\ 0 & 1 & -1 \\ \phi & \delta & \eta \end{pmatrix} \begin{pmatrix} y_t \\ p_t \\ e_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\eta\beta_2}{\beta_1} & \frac{\eta}{\beta_1} & \eta \end{pmatrix} \begin{pmatrix} y_{t-1} \\ p_{t-1} \\ e_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{\eta+c_2}{\beta_1} \end{pmatrix} m_t$$

$$+ \begin{pmatrix} -sE_{t-1}p_t \\ 0 \\ c_2 E_t P_{t+1} + \eta(E_t e_{t+1} - E_{t-1} e_t) - \frac{\eta}{\beta_1} m_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} u_t \\ p_t^* \\ \eta(r_t^* - r_{t-1}^*) + w_t \end{pmatrix} \tag{3.3}$$

where

$$\phi = (1 - c_1) + \frac{\beta_2(\eta + c_2)}{\beta_1}$$

$$\delta = c_2(1 + \frac{1}{\beta_1}) + \frac{\eta}{\beta_1}$$

Let

$$D \equiv \begin{pmatrix} 1 & -s & 0 \\ 0 & 1 & -1 \\ \phi & \delta & \eta \end{pmatrix} \quad \text{for convenience} \tag{3.4}$$

where

$$\begin{aligned} |D| &\equiv \text{the determinant of matrix } D \\ &= \eta + \delta + s\phi > 0 \end{aligned}$$

$$D^{-1} = \frac{1}{|D|} \begin{pmatrix} \eta + \delta & s\eta & s \\ -\phi & \eta & 1 \\ -\phi & (\delta + s\phi) & 1 \end{pmatrix}$$

The model can be expressed as state-space representation:

$$
\begin{pmatrix} y_t \\ p_t \\ e_t \end{pmatrix} = \frac{1}{|D|}\left[\begin{pmatrix} \frac{s\beta_2\eta}{\beta_1} & \frac{s\eta}{\beta_1} & s\eta \\ \frac{\eta\beta_2}{\beta_1} & \frac{\eta}{\beta_1} & \eta \\ \frac{\eta\beta_2}{\beta_1} & \frac{\eta}{\beta_1} & \eta \end{pmatrix}\begin{pmatrix} y_{t-1} \\ p_{t-1} \\ e_{t-1} \end{pmatrix} + \begin{pmatrix} \frac{s(\eta+c_2)}{\beta_1} \\ \frac{\eta+c_2}{\beta_1} \\ \frac{\eta+c_2}{\beta_1} \end{pmatrix} m_t \right.
$$

$$
+ \begin{pmatrix} -s(\eta+\delta)E_{t-1}p_t + sc_2E_tp_{t+1} + s\eta(E_te_{t+1} - E_{t-1}e_t) - \frac{s\eta}{\beta_1}m_{t-1} \\ s\phi E_{t-1}p_t + c_2E_tp_{t+1} + \eta(E_te_{t+1} - E_{t-1}e_t) - \frac{\eta}{\beta_1}m_{t-1} \\ s\phi E_{t-1}p_t + c_2E_tp_{t+1} + \eta(E_te_{t+1} - E_{t-1}e_t) - \frac{\eta}{\beta_1}m_{t-1} \end{pmatrix}
$$

$$
\left. + \begin{pmatrix} (\eta+\delta)u_t + s\eta p_t^* + s\eta(r_t^* - r_{t-1}^*) + sw_t \\ -\phi u_t + \eta p_t^* + \eta(r_t^* - r_{t-1}^*) + w_t \\ -\phi u_t - (\delta + s\phi)p_t^* + \eta(r_t^* - r_{t-1}^*) + w_t \end{pmatrix}\right] \tag{3.5}
$$

Equation (3.5) is rewritten as the usual state-space representation in the following notational form: $x(t) = Ax(t-1) + Bv(t) + c(t) + \xi(t)$ where

$$
A = \frac{1}{|D|}\begin{pmatrix} \frac{s\beta_2\eta}{\beta_1} & \frac{s\eta}{\beta_1} & s\eta \\ \frac{\eta\beta_2}{\beta_1} & \frac{\eta}{\beta_1} & \eta \\ \frac{\eta\beta_2}{\beta_1} & \frac{\eta}{\beta_1} & \eta \end{pmatrix}
$$

$$
B = \frac{(\frac{(\eta+c_2)}{\beta_1})}{|D|}\begin{pmatrix} s \\ 1 \\ 1 \end{pmatrix}
$$

$$
x(t) = \begin{pmatrix} y_t \\ p_t \\ e_t \end{pmatrix} \qquad x(t-1) = \begin{pmatrix} y_{t-1} \\ p_{t-1} \\ e_{t-1} \end{pmatrix}
$$

$$
v(t) = m_t
$$

$c(t)$ is the time-varying nonrandom composite matrix of which the elements are linear combinations of $E_{t-1}p_t$, $E_t p_{t+1}$, and $(E_t e_{t+1} - E_{t-1}e_t)$

$\xi(t)$ is a time-independent random vector having zero mean and a finite constant covariance matrix

$$\xi(t) = \frac{1}{|D|}\begin{pmatrix} (\eta+\delta)u_t + s\eta p^* + s\eta(r_t^* - r_{t-1}^*) + sw_t \\ -\phi u_t + \eta p^* + \eta(r_t^* - r_{t-1}^*) + w_t \\ -\phi u_t - (\delta + s\phi)p^* + \eta(r_t^* - r_{t-1}^*) + w_t \end{pmatrix}$$

The monetary authority is assumed to take economic stability as the main goal and much concerned with the fluctuations of some important macroeconomic variables of the system over time. The objective of the authority is in attempt to minimize the fluctuations of real output , price level, and exchange rate over time by controlling the money supply at each time t. More specifically, the objective function is to minimize the expected value of the weighted total sum of squares of all the state variable deviations from their target values over time.

The following setting of dynamic optimal control method is referred to Murata's theorem[2]. The crucial assumptions are steady-state equilibrium of all the state variables and the serial uncorrelation between all the lagged state variables and random disturbances. In steady-state equilibrium, expectation value of each state variable is convergent to a constant value and the variance of state variable is finite and constant. The constant expectation value and asymptotic variance of state variable turn out to be two groups of the constraints in our dynamic optimal

[2]See Theorem 8 in Chapter 5 of Murata (1982) on pp. 151-153.

control problem. The objective function is set to be linear and quadratic function. The linear and quadratic function has the property of equivalence of certainty.

Assume that the people and govermnemt share the same information set each time. They form their expectations rationally. Their expectations about $t+1$ given the information set at time t, Ω_t, include observed values of current and all the previous endogenous and exogenous variables, $\Omega_t = \{x_t, x_{t-1}, x_{t-2}, ...\}$ where x_t is the vector of the variables at time t.

3.1.1 The derivation of the optimal control law

The following is Murata's theorem (1982). Suppose a quadratic loss function of the form

$$W_0 = \sum_{t=1}^{\infty} \Pi^{t-1} \left[\begin{pmatrix} y_t & p_t & e_t \end{pmatrix} \begin{pmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{pmatrix} \begin{pmatrix} y_t \\ p_t \\ e_t \end{pmatrix} + \begin{pmatrix} y_t & p_t & e_t \end{pmatrix} \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} + (\frac{K_1}{2})^2 + (\frac{K_2}{2})^2 + (\frac{K_3}{2})^2 \right] \quad (3.6)$$

with $K_{ii} > 0$, (i =1, 2, 3), and Π is a time discounted factor, $0 < \Pi < 1$, the quadratic function of W_0 is very close to the following functional form W_1,

$$W_1 = \sum_{t=1}^{\infty} \Pi^{t-1}[K_{11}(y_t - \bar{y})^2 + K_{22}(p_t - \bar{p})^2 + K_{33}(e_t - \bar{e})^2] \quad (3.7)$$

where

$$\bar{y} \equiv (\frac{-K_1}{2K_{11}}), \quad \bar{p} \equiv (\frac{-K_2}{2K_{22}}), \quad \bar{e} \equiv \frac{-K_3}{2K_{33}}$$

$\overline{y}$, $\overline{p}$, $\overline{e}$ are arbitrage target values of real income, price and exchange rate. For convenience, in the proof process, W_1 can be expressed in the matrix representation. The objective is to minimize the expected value of W_1,

$$E(W_1) = E[\sum_{t=1}^{\infty} \Pi^{t-1}(x(t) - a(t))^T \Gamma(x(t) - a(t))] \tag{3.8}$$

subject to the state-space representation

$$x(t) = Ax(t-1) + Bv(t) + c(t) + \xi(t) \tag{3.9}$$

with the known initial value of x(t), x(0), where

$0 < \Pi < 1$

Γ is a symmetric positive semi-definite constant matrix

giving the relative penalties for the squared deviations of the state variables from their targeted values

$$\Gamma = \begin{pmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{pmatrix}, \quad K_{ii} > 0, (i = 1, 2, 3)$$

$x(t)$ is a vector of state variables,

$x^T(t) = (y_t, p_t, e_t)$

$a(t)$ is a vector of the target values for the state vector $x(t)$,

$a^T(x) = (\overline{y}, \overline{p}, \overline{e})$

$x(t-1)$ is a vector of lagged state variables,

$x^T(t-1) = (y_{t-1}, p_{t-1}, e_{t-1})$.

$v(t)$ is a vector of control variables, $v(t) = m_t$.

$c(t)$ is a time-varying nonrandom matrix.

$\xi(t)$ is a matrix associated with random disturbance terms

with zero mean and a finite covariance.

$(\bullet)^T$ is transposition of the matrix.

Using the Lagrangian multiplier approach, we choose $v(t)$ to minimize the expected loss at any time t, but Π^{t-1} can be neglected.

$$E[(x(t) - a(t))^T \Gamma(x(t) - a(t))] \tag{3.10}$$

Assuming a feedback control law of the form

$$v(t) = -Kx(t-1) - k(t) \tag{3.11}$$

Substituting the feedback control law into the constraint of the state-space representation, we obtain

$$x(t) = [A - BK]x(t-1) - (Bk(t) - c(t)) + \xi(t) \tag{3.12}$$

Note that $E(x(t-1)\xi^T(t)) = 0$ by the assumption that the lagged state-variables are serially uncorrelated with random disturbances. Defining the asymptotic expectation of $x(t)$, $E(x(t)) \equiv \mu$, where μ is a constant value, and $E(x(t)x^T(t)) = X$, where $E(x(t)x^T(t)) = cov(x(t)) - \mu\mu^T$, and $cov(x(t))$ is var-cov matrix of $x(t)$, we have

$$\mu = [A - BK]\mu - (Bk(t) - c(t)) \tag{3.13}$$

$$\begin{aligned} X &= [A - BK]X[A - BK]^T + (Bk(t) - c(t))(Bk(t) - c(t))^T \\ &\quad -2[A - BK]\mu(Bk(t) - c(t))^T + R \end{aligned} \tag{3.14}$$

where R is the term of the remainder. The objective equation (3.10) may be reduced to

$$E(x^T(t)\Gamma x(t)) - 2a^T(t)\Gamma Ex(t) = tr(\Gamma X) - 2a^T(t)\Gamma\mu \tag{3.15}$$

since

$$\begin{aligned} E(x^T(t)\Gamma x(t)) &= \mu^T\Gamma\mu + tr(\Gamma cov(x(t))) \\ &= \mu^T\Gamma\mu + tr(\Gamma X) - tr(\Gamma\mu\mu^T) \\ &= tr(\Gamma X) \end{aligned}$$

Now, our problem is simplified to minimize equation (3.14) subject to the constraints of equation (3.13) of asymptotic mean of state variable, $E(x(t))$, and equation (3.14) of $E(x(t)x^T(t))$, which are directly derived from the state-space representation of equation (3.9). Hence the corresponding Lagrangian form is

$$\begin{aligned} L(t) &= tr(\Gamma X) - 2a^T(t)\Gamma\mu \\ &\quad +tr[S\{[A-BK]X[A-BK]^T + (Bk(t)-c(t))(Bk(t)-c(t))^T \\ &\quad +R - X - 2[A-BK]\mu[Bk(t)-c(t)]^T\}] \\ &\quad +2P^T(t)\{[A-BK]\mu[Bk(t)-c(t)] - \mu\} \end{aligned} \tag{3.16}$$

where S is the matrix of Lagrangian multipliers associated with equation (3.14), the constraints of the asymptotic expectation of $E(x(t)x^T(t))$ of equation (3.14). S is assumed to be symmetric. $P(t)$ is the column vector of Lagrangian multipliers associated with the constraint of the asymptotic mean of $x(t)$ of equation (3.13).

The choice variables are X, K, μ, and $k(t)$. We choose X, K, μ, and $k(t)$ to minimize $L(t)$ in equation (3.16). Thus, differentiating equation (3.16) with respect to X, K, μ, and $k(t)$, respectively, and setting the first order conditions equal to zero, we have

$$
\begin{aligned}
(a) \quad & \frac{\partial L(t)}{\partial X} = \Gamma + [A - BK]^T S[A - BK] - S = 0 \\
(b) \quad & \frac{\partial L(t)}{\partial K} = 2\{-[A - BK]^T S(Bk(t) - c(t)) + [A - BK - I]^T P(t) - \Gamma a(t)\} = 0 \\
(c) \quad & \frac{\partial L(t)}{\partial \mu} = 2\{-[A - BK]^T S(Bk(t) - c(t)) + [A - BK - I]^T P(t) - \Gamma a(t)\} \\
(d) \quad & \frac{\partial L(t)}{\partial k(t)} = 2\{B^T S(Bk(t) - c(t)) - B^T S[A - BK]\mu - B^T P(t)\} = 0 \qquad (3.17)
\end{aligned}
$$

Equation (3.17d) is multiplied by μ^T, and then subtracting it from equation (3.17b), we obtain

$$B^T S[A - BK][\mu\mu^T - X] = 0 \qquad (3.18)$$

Under the assumption that $\mu\mu^T - X$ is nonsingular, we get from equation (3.18) that

$$B^T S[A - BK] = 0 \qquad (3.19)$$

so that we have

$$K = (B^T SB)^{-1} B^T SA \qquad (3.20)$$

Defining $L \equiv [A - BK]^T$, from equations (3.17a) and (3.19), we have

$$S = \Gamma + LSA \qquad (3.21)$$

Equations (3.17c) and (3.17d) can be reduced to

$$B^T S(Bk(t) - c(t)) - B^T p(t) = 0 \qquad (3.22)$$

$$[A - BK]^T Sc(t) + [A - BK - I]^T P(t) - \Gamma a(t) = 0 \qquad (3.23)$$

And then from equations (3.22) and (3.23), we have, respectively

$$k(t) = (B^T SB)^{-1} B^T [Sc(t) + P(t)] \tag{3.24}$$

$$P(t) = (I - L)^{-1} [LSc(t) - \Gamma a(t)] \tag{3.25}$$

Substituting equation (3.25) into (3.24), and since $I + (I - L)^{-1} L = (I - L)^{-1}$, we obtain

$$k(t) = (B^T SB)^{-1} B^T [I - L]^{-1} [Sc(t) - \Gamma a(t)] \tag{3.26}$$

From the above proof, the optimal control law is given by

$$v(t) = -Kx(t-1) - k(t), \text{ for } t = 1, 2, \ldots \tag{3.27}$$

where

$$\begin{aligned} K &= (B^T SB)^{-1} B^T SA \\ S &= \Gamma + LSA \\ L &= [A - BK]^T \\ k(t) &= (B^T SB)^{-1} B^T (I - L)^{-1} [Sc(t) - \Gamma a(t)] \end{aligned}$$

The application of the above Murata's theorem to the linear exogenous macroeconomic system of T-B model has the following results:

$$K = \frac{1}{\frac{\eta + c_2}{\beta_1}} \left(\frac{\eta \beta_2}{\beta_1}, \ \frac{\eta}{\beta_1}, \ \eta \right) \tag{3.28}$$

$$L = \underset{\sim}{0}, \quad \underset{\sim}{0} \text{ is a null matrix with zero-element} \tag{3.29}$$

$$S = \begin{pmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{pmatrix} \tag{3.30}$$

$$
\begin{aligned}
k(t) &= \frac{1}{(\frac{\eta+c_2}{\beta_1})(s^2K_{11}+K_{22}+K_{33})} * \\
&\{sK_{11}[-s(\eta+\delta)E_{t-1}p_t + sc_2E_tp_{t+1} + s\eta(E_te_{t+1} \\
&E_{t-1}e_t) - \frac{s\eta}{\beta_1}m_{t-1} - \bar{y}|D|] \\
&+K_{22}[s\phi E_{t-1}p_t + c_2E_tp_{t+1} + \eta(E_te_{t+1} - E_{t-1}e_t) \\
&-\frac{\eta}{\beta_1}m_{t-1} - \bar{p}|D|] \\
&+K_{33}[s\phi E_{t-1}p_t + c_2E_tp_{t+1} + \eta(E_te_{t+1} - E_{t-1}e_t) \\
&-\frac{\eta}{\beta_1}m_{t-1} - \bar{e}|D|]\} \qquad (3.31) \\
\gamma(t) &= -k(t) \\
&= \frac{\eta}{\eta+c_2}m_{t-1} + \gamma_0 + \gamma_1E_{t-1}p_t + \gamma_2E_tp_{t+1} + \\
&\gamma_3(E_te_{t+1} - E_{t+1}e_t) \qquad (3.32)
\end{aligned}
$$

where

$$\gamma_0 = \frac{(sK_{11}\bar{y} + K_{22}\bar{p} + K_{33}\bar{e})}{(\frac{\eta+c_2}{\beta_1})(s^2K_{11}+K_{22}+K_{33})}$$

$$\gamma_1 = \frac{[s^2(\eta+\delta)K_{11} - s\phi K_{22} - s\phi K_{33}]}{(\frac{\eta+c_2}{\beta_1})(s^2K_{11}+K_{22}+K_{33})} \gtrless 0$$

$$\gamma_2 = \frac{-c_2}{(\frac{\eta+c_2}{\beta_1})} < 0$$

$$\gamma_3 = \frac{-\eta}{(\frac{\eta+c_2}{\beta_1})} < 0$$

Substituting the values of K in equation (3.28) and $\gamma(t)$ in equation (3.32) into the optimal control law, we have the optimal feedback money supply rule

$$
\begin{aligned}
m_t &= \frac{-\eta\beta_2}{(\eta+c_2)}y_{t-1} - \frac{\eta}{(\eta+c_2)}p_{t-1} - \frac{\eta\beta_1}{(\eta+c_2)}e_{t-1} + \frac{\eta}{(\eta+c_2)}m_{t-1} \\
&+\gamma_0 + \gamma_1E_{t-1}P_t + \gamma_2E_tP_{t+1} + \gamma_3(E_te_{t+1} - E_{t-1}e_t) \qquad (3.33)
\end{aligned}
$$

Substituting equation (3.33) into state-space representation (3.5), we have the pseudo-reduced forms of state variables, real output, price, and exchange rate under the optimal control of money supply

$$y_t = s\alpha_0 + \alpha_1 E_{t-1}P_t + \xi_{y,t} \tag{3.34}$$

$$p_t = \alpha_0 + \alpha_2 E_{t-1}P_t + \xi_{p,t} \tag{3.35}$$

$$e_t = \alpha_0 + \alpha_2 E_{t-1}P_t + \xi_{e,t} \tag{3.36}$$

where

$$\alpha_0 = \frac{sK_{11}\bar{y} + K_{22}\bar{p} + K_{33}\bar{e}}{s^2K11 + K_{22} + K_{33}} > 0$$

$$\alpha_1 = \frac{-s(K_{22} + K_{33})}{s^2K_{11} + K_{22} + K_{33}} < 0$$

$$\alpha_2 = \frac{s^2K_{11}}{s^2K_{11} + K_{22} + K_{33}}$$

$$\xi_{y,t} = \frac{1}{|D|}[(\eta + \delta)u_t + s\eta p_t^* + s\eta(r_t^* - r_{t-1}^*) + sw_t]$$

$$\xi_{p,t} = \frac{1}{|D|}[-\phi u_t + \eta p_t^* + \eta(r_t^* - r_{t-1}^*) + w_t]$$

$$\xi_{e,t} = \frac{1}{|D|}[-\phi u_t - (\delta + s\phi)p_t^* + \eta(r_t^* - r_{t-1}*) + w_t]$$

Taking conditional expectation on both sides of equation (3.35), we solve for the solution of rational expectation of p_t

$$E_{t-1}p_t = \frac{\alpha_0}{1 - \alpha_2} \tag{3.37}$$

Then, substituting the solutions of $E_{t-1}p_t$, E_tP_{t+1}, $E_{t-1}e_t$, and E_te_{t+1} into equation (3.33), the final form of the optimal money supply rule can be simply expressed as

$$m_t = C - \frac{\eta\beta_2}{\eta + c_2}y_{t-1} - \frac{\eta}{\eta + c_2}p_{t-1} - \frac{\eta\beta_1}{\eta + c_2}e_{t-1} + \frac{\eta}{\eta + c_2}m_{t-1} \tag{3.38}$$

where C is a constant, $C = \gamma_0 + [\alpha_0/(1 - \alpha_2)](\gamma_1 + \gamma_2)$. The control rule is deterministic and involves no surprise. Importantly, it includes a constant term which captures all the expectation values of e_t and p_t.

From equation (3.38), the optimal feedback money supply rule only depends on the previous real output, price, interest rate and money supply. In other words, the deterministic feedback money supply rule is feedback from past target variables and policy instruments. According to the guidance of the optimal feedback money supply rule, the monetary authority should decrease the current money supply when the previous exchange rate increased last year. Since the exchange rate increased last year, the domestic agents would be induced to sell foreign bonds during the previous year. Given the expectation of the future exchange rate, net purchase of the foreign bonds increase instantaneously. Capital outflow has the potential impact on the exchange depreciation. Therefore, in order to achieve economic stability, money supply is required to decrease to offset the force towards exchange depreciation arising from capital outflow[3].

From the derived optimal control law, the monetary authority should decrease money supply to offset the force towards price increase. On the other hand, when the interest rate increases, it would induce the domestic agent to sell foreign bonds and purchase the domestic bonds. Capital inflow would have impact on the appreciation of exchange rate.

Similarly, the final forms of the state variables, real output, price, and exchange rate, under the optimal money supply rule and rational expectation of price and

[3]Under flexible exchange rate system, increases in money supply would cause the depreciation of exchange rate. It would possibly result in more variation of exchange rate and price level.

exchange rate, can be easily obtained by substituting the solution of equation (3.37) into equations (3.34)-(3.36). Then, we have the reduced-forms of output, price and exchange rate under our derived money supply rule,

$$y_t = \xi_{y,t} \tag{3.39}$$

$$p_t = \frac{\alpha_0}{1-\alpha_2} + \xi_{p,t} \tag{3.40}$$

$$e_t = \frac{\alpha_0}{1-\alpha_2} + \xi_{e,t} \tag{3.41}$$

where

$$\frac{\alpha_0}{1-\alpha_2} = \frac{sK_{11}\bar{y} + K_{22}\bar{p} + K_{33}\bar{e}}{K_{22}+K_{33}} > 0$$

The income, price and exchange rate become exogenous processes under the derived optimal control rule of money supply. The weights of the penalties given to the deviations of all the target variables in the loss function do not enter into the income exogenous process. But the weights only influence the time trend of the price and exchange rate processes.

From equations (3.39)-(3.41) and (2.2), the final form of interest rate is

$$\begin{aligned} r_t &= \beta_0 + \frac{\eta\beta_2}{\beta_1(\eta+c_2)}y_{t-1} + \frac{\eta}{\beta_1(\eta+c_2)}p_{t-1} + \frac{\eta}{(\eta+c_2)}e_{t-1} \\ &\quad -\frac{\eta}{\beta_1(\eta+c_2)}m_{t-1} + \xi_{r,t} \end{aligned} \tag{3.42}$$

where

$$\begin{aligned} \beta_0 &\ \text{ is a constant,} \\ \beta_0 &= \frac{1}{\beta_1}[(\frac{\alpha_0}{1-\alpha_2})(1-\gamma_1-\gamma_2)-\gamma_0] \\ \xi_{r,t} &= \frac{1}{|D|}\{\frac{1}{\beta_1}[\beta_2(\eta+\delta)-\phi]u_t + \frac{\eta}{\beta_1}(1+s\beta_2)p_t^* \\ &\quad +\frac{\eta}{\beta_1}(1+s\beta_2)(r_t^*-r_{t-1}^*) + \frac{1}{\beta_2}(1+s\beta_2)w_t\} \end{aligned}$$

Under our derived money supply rule, the interest rate depends on the lagged policy instrument of money supply, and the lagged endogenous variables which are targeted in the government objective function. Unlike real income, price and exchange rate, the interest rate is autoregressive process, rather than exogenous process.

3.1.2 The asymptotic variances of state variables

The relationship between the variance of state variable and those of random disturbances can be calculated from the final form of the state variable. From equations (3.39)-(3.41), we have that

$$\sigma_y^2 = \frac{1}{|D|^2}[(\eta+\delta)^2\sigma_u^2 + (s\eta)^2\sigma_{p*}^2 + 2(s\eta)^2\sigma_{r*}^2 + s^2\sigma_w^2] \quad (3.43)$$

$$\sigma_p^2 = \frac{1}{|D|^2}[\phi^2\sigma_u^2 + \eta^2\sigma_{p*}^2 + 2\eta^2\sigma_{r*}^2 + \sigma_w^2] \quad (3.44)$$

$$\sigma_e^2 = \frac{1}{|D|^2}[\phi^2\sigma_u^2 + (\delta+s\phi)^2\sigma_{p*}^2 + 2\eta^2\sigma_{r*}^2 + \sigma_w^2] \quad (3.45)$$

Since random disturbances are serially and contemporaneously uncorrelated, the variances of the variables, y_t, p_t, e_t, are simply linear combinations of variances of all the random disturbances. The variations of all these variables only attribute to the four random disturbances: domestic random supply innovation u_t, foreign price disturbance p_t^*, foreign interest rate disturbance r_t^*, and domestic demand disturbance w_t. All the target variables under the derived optimal money supply rule and with rational expectations turn out to be exogenous processes and the lagged state variables do not enter as arguments in the determination of these target variables.

3.1.3 The effects of random disturbances

We proceed to analyze the impacts of the disturbances on the domestic economy under rational expectations and monetary authority optimal control of money supply. The impact on all the endogenous variables can be easily seen from the partial derivatives with respect to the random disturbances. Here, we only discuss the effect on the target variables (y, p, e). In addition, the role of the degree of capital mobility in the variations of the endogenous variables would be worthy to analyze for an open economy later on.

(1) foreign price disturbance

Taking partial differentiation of final forms of y_t, p_t, and e_t, with respect to p_t^*, respectively

$$\frac{\partial y_t}{\partial p_t^*} = \frac{s\eta}{|D|} > 0 \qquad (3.46)$$

$$\frac{\partial p_t}{\partial p_t^*} = \frac{\eta}{|D|} > 0 \qquad (3.47)$$

$$\frac{\partial e_t}{\partial p_t^*} = \frac{-(\delta + s\phi)}{|D|} < 0 \qquad (3.48)$$

The unanticipated increases in foreign price have brought into real output and price level increases but exchange rate decreases. Given the initial exchange rate and expectation of exchange rate, increases in foreign price would cause domestic price increase and unanticipated price increase, and lead to domestic aggregate supply increase. And given money supply, increases in real output and price would lead to demand for money increase. Then, the interest rate is required to increase in order to attain money market equilibrium. Furthermore, increases in domestic interest rate induce domestic agents selling foreign bonds and purchasing domestic bonds.

Capital inflow will lead to the appreciation of exchange rate.

(2) foreign interest rate disturbance

Taking partial derivative of final forms of y_t, p_t, e_t, with respect to foreign interest rate, r_t^*, we have

$$\frac{\partial y_t}{\partial r_t^*} = \frac{1}{|D|} s\eta > 0 \tag{3.49}$$

$$\frac{\partial p_t}{\partial r_t^*} = \frac{1}{|D|} \eta > 0 \tag{3.50}$$

$$\frac{\partial e_t}{\partial r_t^*} = \frac{1}{|D|} \eta > 0 \tag{3.51}$$

The foreign interest rate increases cause real output, price and exchange rate increases. The random disturbance of foreign interest rate has the same effect on price and exchange rate. Given the expectation of exchange rate, the increases in foreign interest rate induce domestic agents to purchase more foreign bonds. Net position of the domestic economy in foreign bonds would therefore increase. The capital outflow causes depreciation of exchange rate. Consequently, domestic price level will increase to maintain the purchasing power parity. The unanticipated increases in price will boost the real output.

(3) domestic supply disturbance

Similarly, taking partial differentiation of final forms of y_t, p_t, and e_t, with respect to domestic supply disturbance, u_t, it is easily obtained

$$\frac{\partial y_t}{\partial u_t} = \frac{\eta + \delta}{|D|} > 0 \tag{3.52}$$

$$\frac{\partial p_t}{\partial u_t} = \frac{-\phi}{|D|} < 0 \tag{3.53}$$

$$\frac{\partial e_t}{\partial u_t} = \frac{-\phi}{|D|} < 0 \tag{3.54}$$

The unanticipated domestic supply shock causes real output increase but price level and exchange rate decrease. It has the same direction and size effects on price and exchange rate. Given the expectation of price, the unanticipated domestic supply innovation results in aggregate real output increases and therefore price level decreases. Given the initial money supply, both real income and price increases will lead to demand for money increases. It follows that the interest rate will increase to maintain money market equilibrium leading the capital inflow, and consequently, the appreciation of exchange rate.

(4) domestic demand disturbance

The partial derivatives of final forms of y_t, p_t, and e_t, with respect to domestic demand disturbance, w_t, are

$$\frac{\partial y_t}{\partial w_t} = \frac{s}{|D|} > 0 \tag{3.55}$$

$$\frac{\partial p_t}{\partial w_t} = \frac{1}{|D|} > 0 \tag{3.56}$$

$$\frac{\partial e_t}{\partial w_t} = \frac{1}{|D|} > 0 \tag{3.57}$$

The unanticipated increases in domestic demand will result in real output, price and exchange rate increases. It has the same direction and size of the effect on price and exchange rate. The interpretation of these results is analogous to the above.

3.1.4 The degree of capital mobility

How the degree of capital mobility is related to the fluctuations of the endogenous variables originating from a variety of random disturbances. Whether the more capital mobility would enforce or lesson the variations of the endogenous variables depends on the origin of random disturbance.

(1) foreign price disturbance

The partial derivatives of the absolute values of

$$\frac{\partial y_t}{\partial p_t^*}, \quad \frac{\partial p_t}{\partial p_t^*}, \text{ and } \frac{\partial e_t}{\partial p_t^*}$$

with respect to the measure of the degree of capital mobility, η, are

$$\frac{\partial |\frac{\partial y_t}{\partial p_t^*}|}{\partial \eta} = \frac{s[s(1-c_1)+c_2(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})]}{|D|^2} > 0 \tag{3.58}$$

$$\frac{\partial |\frac{\partial p_t}{\partial p_t^*}|}{\partial \eta} = \frac{[s(1-c_1)+c_2(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})]}{|D|^2} > 0 \tag{3.59}$$

$$\frac{\partial |\frac{\partial e_t}{\partial p_t^*}|}{\partial \eta} = \frac{[s(1-c_1)+c_2(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})]}{|D|^2} > 0 \tag{3.60}$$

As the degree of the capital mobility increases, the fluctuations of real output, and price level originating from unanticipated disturbance of foreign price will increase. But the fluctuation of the exchange rate which arises from the foreign price shock will be reduced as capital mobility increases.

(2) foreign interest rate disturbance

The partial derivatives of the absolute values of

$$\frac{\partial y_t}{\partial r_t^*}, \quad \frac{\partial p_t}{\partial r_t^*}, \text{and} \frac{\partial e_t}{\partial r_t^*},$$

with respect to the measure of the degree of capital mobility, η, are

$$\frac{\partial |\frac{\partial y_t}{\partial r_t^*}|}{\partial \eta} = \frac{s[s(1-c_1)+c_2(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})]}{|D|^2} > 0 \tag{3.61}$$

$$\frac{\partial |\frac{\partial p_t}{\partial r_t^*}|}{\partial \eta} = \frac{[s(1-c_1)+c_2(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})]}{|D|^2} > 0 \tag{3.62}$$

$$\frac{\partial|\frac{\partial e_t}{\partial r_t^*}|}{\partial \eta} = \frac{[s(1-c_1)+c_2(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})]}{|D|^2} > 0 \tag{3.63}$$

As the degree of capital mobility increases, the fluctuations of real output, price level and exchange rate originating from the fluctuations of foreign interest rate will be increased. It implies that the higher capital mobility enlarges the variation of domestic economy which arises from the fluctuation of foreign interest rate.

(3) domestic supply disturbance

The partial derivatives of the absolute values of

$$\frac{\partial y_t}{\partial u_t}, \quad \frac{\partial p_t}{\partial u_t}, \text{ and } \frac{\partial e_t}{\partial u_t}$$

with respect to the measure of the degree of capital mobility, η, are

$$\frac{\partial|\frac{\partial y_t}{\partial u_t}|}{\partial \eta} = \frac{s(1-c_1)(1+\frac{1}{\beta_1})}{|D|^2} > 0 \tag{3.64}$$

$$\frac{\partial|\frac{\partial p_t}{\partial u_t}|}{\partial \eta} = \frac{-(1-c_1)(1+\frac{1}{\beta_1})}{|D|^2} < 0 \tag{3.65}$$

$$\frac{\partial|\frac{\partial e_t}{\partial u_t}|}{\partial \eta} = \frac{-(1-c_1)(1+\frac{1}{\beta_1})}{|D|^2} < 0 \tag{3.66}$$

As the degree of capital mobility increase, the fluctuations of real output originating from fluctuations of domestic supply will increase, but price and exchange rate fluctuations originating from domestic supply disturbances will decrease. The degree of capital mobility appears to have the functioning in smoothing the variations of nominal monetary variables, such as price level and exchange rate from domestic supply shock.

(4) domestic demand disturbance

The partial derivatives of the absolute values of

$$\frac{\partial y_t}{\partial w_t}, \frac{\partial p_t}{\partial w_t}, \text{ and} \frac{\partial e_t}{\partial w_t}$$

with respect to the measure of the degree of capital mobility, η, are

$$\frac{\partial |\frac{\partial y_t}{\partial w_t}|}{\partial \eta} = \frac{-s(1 + \frac{1}{\beta_1} + \frac{s\beta_2}{\beta_1})}{|D|^2} < 0 \tag{3.67}$$

$$\frac{\partial |\frac{\partial p_t}{\partial w_t}|}{\partial \eta} = \frac{-(1 + \frac{1}{\beta_1} + \frac{s\beta_2}{\beta_1})}{|D|^2} < 0 \tag{3.68}$$

$$\frac{\partial |\frac{\partial e_t}{\partial w_t}|}{\partial \eta} = \frac{-(1 + \frac{1}{\beta_1} + \frac{s\beta_2}{\beta_1})}{|D|^2} < 0 \tag{3.69}$$

As the degree of capital mobility increases, the fluctuations of all the state variables originating from domestic demand disturbance will increase. The high capital mobility appears to insulate the domestic economy from domestic demand disturbances.

From the above analysis of the effect of capital mobility on the variation of domestic economy, we have the conclusion that higher capital mobility enlarges the variations of domestic economy which result from disturbances of foreign interest rate, domestic supply, and foreign price. Conversely, it has insulated the variations of domestic economy which result from domestic demand disturbances.

3.2 One Target Variable Case

In this section, we consider the situation of only one target variable, real output y_t. It is assumed that government is only concerned with the business cycle or the fluctuations of real output over time. The objective function is to minimize the expectation value of the sum of squares of real output deviations from its targeted value or trend level, $\overline{y}$, *i.e.*,

$$\min E \sum_{t=1}^{\infty} \Pi^{t-1}(y_t - \overline{y})^2 \tag{3.70}$$

Substitution of p_t and p_{t-1} from equation (3.2a), r_t and r_{t-1} from equation (3.2b), e_t and e_{t-1} from equation (3.2c) into equation (3.2d), then the model can be reduced to

$$\begin{aligned}
&\{(1-c_1) + (\eta + c_2)[\tfrac{1}{s}(1+\tfrac{1}{\beta_1}) + \tfrac{\beta_2}{\beta_1}]\}y_t \\
&= \eta[\tfrac{1}{s}(1+\tfrac{1}{\beta_1}) + \tfrac{\beta_2}{\beta_1}]y_{t-1} + (\tfrac{\eta+c_2}{\beta_1})m_t - \tfrac{\eta}{\beta_1}m_{t-1} \\
&-(\eta+c_2)(1+\tfrac{1}{\beta_1})E_{t-1}p_t + \eta(1+\tfrac{1}{\beta_1})E_{t-2}p_{t-1} + c_2E_tp_{t+1} \\
&+\eta(E_te_{t+1} - E_{t-1}e_t) + \tfrac{\eta}{s}(p_t^* - p_{t-1}^*) + \tfrac{\eta}{s}(r_t^* - r_{t-1}^*) \\
&+(\tfrac{\eta+c_2}{s})(1+\tfrac{1}{\beta_1})u_t - \tfrac{\eta}{s}(1+\tfrac{1}{\beta_1})u_{t-1} + w_t
\end{aligned} \tag{3.71}$$

Let

$$(1-c_1) + (\eta + c_2)[\frac{1}{s}(1+\frac{1}{\beta_1}) + \frac{\beta_2}{\beta_1}] = \alpha$$

equation (3.71) can be expressed as state-space representation:

$$y_t = Hy_{t-1} + Jm_t + d(t) + \xi(t) \tag{3.72}$$

where

$$\begin{aligned}
H &= \frac{\eta[\frac{1}{s}(1+\frac{1}{\beta_1}) + \frac{\beta_2}{\beta_1}]}{\alpha} \\
J &= \frac{\eta + c_2}{\beta_1\alpha} \\
d(t) &= \frac{-\eta}{\alpha\beta_1}m_{t-1} - \frac{(\eta+c_2)}{\alpha}(1+\frac{1}{\beta_1})E_{t-1}p_t + \frac{\eta}{\alpha}(1+\frac{1}{\beta_1})E_{t-2}p_{t-1}
\end{aligned}$$

$$
\begin{aligned}
& +\frac{c_2}{\alpha}E_t p_{t+1} + \frac{\eta}{\alpha}(E_t e_{t+1} - E_{t-1}e_t) \\
\xi(t) \;\; = \;\; & \frac{\eta}{s\alpha}(p_t^* - p_{t-1}^*) + \frac{\eta}{s\alpha}(r_t^* - r_{t-1}^*) + \frac{(\eta + c_2)}{s\alpha}(1 + \frac{1}{\beta_1})u_t \\
& -\frac{\eta}{s\alpha}(1 + \frac{1}{\beta_1})u_{t-1} + \frac{w_t}{\alpha}
\end{aligned}
$$

Similarly, the application of the theorem to the model with only one target variable, real output y_t, provides the optimal money supply rule

$$m_t = \frac{-H}{J}y_{t-1} - \frac{d(t)}{J} + \frac{\overline{y}}{J} \tag{3.73}$$

We substitute the derived optimal feedback money supply rule of equation (3.73) into the state-space representation of equation (3.72), then the final form of real output, y_t, is

$$y_t = \overline{y} + \xi(t) \tag{3.74}$$

Under the derived optimal money supply rule and with rational expectation of price and exchange rate, the real output turns out to be determined by the targeted value and a composite random disturbance term which is linear combinations of all the exogenous random disturbances. The real income process does not include the lagged real income. Noted that the income process is exogenous even if the money supply is stationary random process.

The variance of real output, σ_y^2, is easily obtained from the final form of real output of equation (3.74)

$$\sigma_y^2 = \sigma_\xi^2 \tag{3.75}$$

Since all the random variable disturbances are serially and contemporaneously uncorrelated, the variance of real output is

$$\sigma_y^2 = 2(\frac{\eta}{s\alpha})^2\sigma_{p^*}^2 + 2(\frac{\eta}{s\alpha})^2\sigma_{r^*}^2 + \frac{(1 + \frac{1}{\beta_1})^2}{s^2\alpha^2}[(\eta + c_2)^2 + \eta^2]\sigma_u^2 + \frac{\sigma_w^2}{\alpha^2} \tag{3.76}$$

3.2.1 The effects of random disturbances

Following the analysis in the previous section, the analysis of impacts of the disturbances on the domestic economy with rational expectation and under the optimal control of money supply. The impact of every random disturbance on the real output can be easily seen from the partial derivatives of real output with respect to the random disturbances.

The partial derivatives of the final form of real output, y_t, with respect to p_t^*, r_t^*, u_t, and w_t, are

$$\frac{\partial y_t}{\partial p_t^*} = \frac{\eta}{s\alpha} > 0 \tag{3.77}$$

$$\frac{\partial y_t}{\partial r_t^*} = \frac{\eta}{s\alpha} > 0 \tag{3.78}$$

$$\frac{\partial y_t}{\partial u_t} = \frac{(\eta + c_2)}{s\alpha}(1 + \frac{1}{\beta_1}) > 0 \tag{3.79}$$

$$\frac{\partial y_t}{\partial w_t} = \frac{1}{\alpha} > 0 \tag{3.80}$$

From the above partial derivatives of y_t with respect to all the random disturbances, the signs of all the impacts on real output can be determined. All the increases in unanticipated random disturbances can lead to the real output increases with the same results as the three target variable case. Now, the relative insulation of the economy from the random shocks in the alternative cases of three target variables and one target variable are analyzed from the responses of y_t with respect to all the random disturbances. The partial derivatives of y_t with respect to all the random disturbances in these two cases are summarized in Table 3.1.

Some important conclusions can be drawn from the relative magnitudes of partial derivatives with respect to the random disturbances.

Table 3.1: The response of y_t to the random disturbances

	one target variable (y_t)	three target variables (y_t, p_t, e_t)
p_t^*	$\eta/(s\alpha) = \eta/\lvert D\rvert$	$s\eta/\lvert D\rvert$
r_t^*	$\eta/(s\alpha) = \eta/\lvert D\rvert$	$s\eta/\lvert D\rvert$
u_t	$[(\eta + c_2)/(s\alpha)](1 + 1/\beta_1)$	$[(\eta + c_2)/(s\alpha)](1 + 1/\beta_1)$
w_t	$1/\alpha$	$1/\alpha$
	$\alpha = (1 - c_1) + (\eta + c_2)[(1 + 1/\beta_1)/s$ $+\beta_2/\beta_1]$	$\lvert D\rvert = \eta + \delta + s\phi$ $\phi = (1 - c_1) + c_2\beta_2/\beta_1 + \eta\beta_2/\beta_1$ $\delta = c_2(1 + 1/\beta_1) + \eta/\beta_1$

First, the fluctuations of real output originating from the random disturbances of foreign price and foreign interest rate in the case of one target variable is smaller than those in the case of three target variable ($(\eta/(s\alpha) < (s\eta)/(\eta + \delta + s\phi)$, when $s > 1$). It appears that the case of one target variable(real income) with optimal control setting is superior to the case of three target variables if the government chooses stabilization policy based on the minimum variance of output. The above analysis provides the policy implication that for an economy with high elasticity of aggregate supply to unanticipated changes in domestic price, or say with a slight trade-off between unemployment and unanticipated inflation rate along Phillips Curve, monetary authority seems better only consider the real output fluctuations in the setting of objective function. In this way, it may lessen the impacts of random shocks from foreign origins on the real output fluctuations of the domestic economy. On the contrary, for an economy with a low elasticity of aggregate supply with respect to unanticipated changes in domestic price, monetary authority seems appropriate to take into account other endogenous variables such as domestic price and exchange rate, in the loss function in the choice of stabilization policy, in attempt to reduce the impact of random foreign shocks on domestic real output.

Second, the partial derivatives of real output with respect to the corresponding domestic random shocks are the same in the two cases respectively. The fluctuations of real output originating from domestic random shocks is the same in the two cases. It provides policy implication that the impact of domestic random shock on real output is the same irrespective of the government objective function of stabilization policy with only real output or more other variables as target variables.

3.3 The Alternative Money Supply Rule - Fixed Money Supply Rule

In this section, we consider the fixed money supply which is advocated by monetarists such as Friedman. He argues that there should be no use of active countercyclical monetary policy, and that monetary policy should be confined to making the money supply growing at a constant rate. Friedman advocates a simple monetary rule in which the Fed does not respond to the condition of the economy and can make a major contribution to promoting economic stability. Our attempt here is to compare the relative performance of the alternative feedback and fixed money supply rules according to the criteria of smaller variances of real output, price and exchange rate.

Suppose the monetary authority set the simple money supply rule as

$$m_t = g, \ \forall t$$

where g is a constant. By taking conditional expectations of equations (2.9)-(2.12) for an arbitrage time $t+i$ given at time $t-1$, we obtain some important properties of rational expectation solutions of the model.

$$E_{t-1}y_{t+i} = 0, \quad i = 0, 1, 2, \ldots \tag{3.81}$$

$$E_{t-1}p_{t+i} - g = \beta_1 E_{t-1}r_{t+i}, \quad i = 0, 1, 2, \ldots \tag{3.82}$$

$$E_{t-1}p_{t+i} = E_{t-1}e_{t+i}, \quad i = 0, 1, 2, \ldots \tag{3.83}$$

From equation (3.2d), we have

$$\begin{aligned} &\eta[(E_{t-1}e_{t+i+1} - E_{t-1}e_{t+i}) - (E_{t-1}e_{t+i} - E_{t-1}e_{t+i-1}) \\ &\qquad -(E_{t-1}r_{t+i} - E_{t-1}r_{t+i-1})] \\ &= c_2[E_{t-1}r_{t+i} - (E_{t-1}e_{t+i+1} - E_{t-1}e_{t+i})], \quad i = 1, 2, \ldots \end{aligned} \tag{3.84}$$

Since $E_{t-1}r_{t+i} = (1/\beta_1)E_{t-1}e_{t+i} - (g/\beta_1)$, for i=1, 2, 3, ...,
equation (3.84) can be written as

$$\begin{aligned} &\eta[(E_{t-1}e_{t+i+1} - E_{t-1}e_{t+i}) - (E_{t-i}e_{t+i} - E_{t-1}e_{t+i-1}) \\ &\qquad -\frac{1}{\beta_1}(E_{t-1}e_{t+i} - E_{t-1}e_{t+i-1})] \\ &= c_2[\frac{1}{\beta_1}(E_{t-1}e_{t+i} - g) - (E_{t-1}e_{t+i+1} - E_{t-1}e_{t+i})], \quad i = 1, 2, \ldots \end{aligned} \tag{3.85}$$

Rearranging equation (3.85), it can be rewritten as a second-order nonhomogeneous difference equation in $E_{t-1}e_{t+i}$

$$\begin{aligned} &(\eta + c_2)E_{t-1}e_{t+i+1} - [\eta + (\eta + c_2)(1 + \frac{1}{\beta_1})]E_{t-1}e_{t+i} \\ &\qquad +\eta(1 + \frac{1}{\beta_1})E_{t-1}e_{t+i-1} = \frac{-c_2 g}{\beta_1}, \quad i = 1, 2, \ldots \end{aligned} \tag{3.86}$$

The characteristic equation of equation (3.86) is

$$(\eta + c_2)\lambda^2 - [\eta + (\eta + c_2)(1 + \frac{1}{\beta_1})]\lambda + \eta(1 + \frac{1}{\beta_1}) = 0 \tag{3.87}$$

The two roots of characteristic equation (3.87), λ_1 and λ_2 can be solved as

$$\lambda_1 = \frac{\eta}{(\eta + c_2)} < 1, \quad \lambda_2 = 1 + \frac{1}{\beta_1} > 1$$

The complementary solution to equation (3.86) is

$$E_{t-1}e^c_{t+i} = A_1(t-1)\lambda_1^i + A_2(t-1)\lambda_2^i \quad i = 0, 1, 2, \ldots \tag{3.88}$$

or

$$E_{t-1}e^c_{t+i} = A_1(t-1)(\frac{\eta}{\eta + c_2})^i + A_2(t-1)(1 + \frac{1}{\beta_1})^i \quad i = 0, 1, 2, \ldots$$

where $A_1(t-1)$ and $B(t-1)$ are arbitrage initial constants. Since the characteristic root, $\lambda_2 > 1$, the system exhibits saddle-point instability. In order to rule out the explosive system, it is required to make the assumption in the model that $A_2(t-1) = 0$[4]. Therefore, the complementary solution is

$$E_{t-1}e^c_{t+i} = A_1(t-1)(\frac{\eta}{\eta + c_2})^i, \quad i = 0, 1, 2, \ldots \tag{3.89}$$

In addition, the particular solution to equation (3.86) is

$$E_{t-1}e^p_{t+i} = g \tag{3.90}$$

The general solution to nonhomogeneous difference equation is the adding up of the complementary solution and the particular solution. Thus, the general solution to equation (3.86) is

$$E_{t-1}e_{t+i} = A_1(t-1)(\frac{\eta}{\eta + c_2})^i + g \tag{3.91}$$

[4]If $A_2(t-1) \neq 0$, the solution to (3.88) will be explosive.

By taking conditional expectation of equation (3.2d) given at time $t-1$, and since $E_{t-1}r_t = \frac{1}{\beta_1}(E_{t-1}e_t - g)$, we have

$$\eta[(E_{t-1}e_{t+1} - E_{t-1}e_t) - (E_{t-1}e_t - e_{t-1}) - \frac{1}{\beta_1}(E_{t-1}e_t - g) - r^*_{t-1} + r_{t-1}]$$
$$= c_2[\frac{1}{\beta_1}(E_{t-1}e_t - g) - (E_{t-1}e_{t+1} - E_{t-1}e_t)] \qquad (3.92)$$

or

$$(\eta + c_2)E_{t-1}e_{t+1} - [\eta + (\eta + c_2)(1 + \frac{1}{\beta_1})]E_{t-1}e_t$$
$$= -\eta(e_{t-1} - r^*_{t-1} + r_{t-1}) - \frac{g}{\beta_1}(\eta + c_2)$$

The constant $A_1(t-1)$ can now be obtained by substituting the equation (3.91) for $i = 0, 1$, into (3.92) and is given by

$$A_1(t-1) = (\frac{\eta}{\eta + c_2})(\frac{\beta_1}{1+\beta_1})(e_{t-1} - r^*_{t-1} + r_{t-1}) - g(\frac{\eta}{\eta + c_2})(\frac{\beta_1}{1+\beta_1}) \qquad (3.93)$$

Substituting $A_1(t-1)$ into (3.91), we have that

$$E_{t-1}e_{t+i} = E_{t-1}p_{t+i} = (\frac{\eta}{\eta + c_2})^{i+1}(\frac{\beta_1}{1+\beta_1})(e_{t-1} - r^*_{t-1} + r_{t-1})$$
$$+g[1 - (\frac{\eta}{\eta + c_2})^{i+1}(\frac{\beta_1}{1+\beta_1})], \quad i = 0, 1, 2, \ldots \qquad (3.94)$$

Therefore, it is obtained from equation (3.94) that

$$E_{t-1}e_t = E_{t-1}p_t$$
$$= (\frac{\eta}{\eta + c_2})(\frac{\beta_1}{1+\beta_1})(e_{t-1} - r^*_{t-1} + r_{t-1}) - (1 - \frac{\lambda_1}{\lambda_2})g$$
$$= \frac{\lambda_1}{\lambda_2}(e_{t-1} - r^*_{t-1} + r_{t-1}) - -(1 - \frac{\lambda_1}{\lambda_2})g \qquad (3.95)$$

or

$$E_t e_{t+1} = (\frac{\eta}{\eta + c_2})(\frac{\beta_1}{1+\beta_1})(e_t - r^*_t + r_t) - (1 - \frac{\lambda_1}{\lambda_2})g$$
$$= \frac{\lambda_1}{\lambda_2}(e_t - r^*_t + r_t) - (1 - \frac{\lambda_1}{\lambda_2})g$$

For convenience, let $z_t = e_t - r_t^* + r_t$. By the solutions of $E_t e_{t+1}$ and $E_{t-1} e_t$ of equation (3.95), the structure of the model can be reduced to

$$\begin{pmatrix} 1 & 0 & \phi(1-c_1) \\ 0 & -s & 1 \\ \beta_1 & -(1+\beta_1) & -\beta_2 \end{pmatrix} \begin{pmatrix} z_t \\ e_t \\ y_t \end{pmatrix}$$

$$= \begin{pmatrix} \lambda_1 z_{t-1} + \phi[w_t - c_2(p_t^* + r_t^*)] - (\frac{c_2}{\eta + c_2})g \\ -s\frac{\lambda_1}{\lambda_2} z_{t-1} + u_t + sp_t^* + (1 - \frac{\lambda_1}{\lambda_2})g \\ p_t^* - \beta_1 r_t^* - g \end{pmatrix} \tag{3.96}$$

where the determinant of coefficient matrix D, and ϕ are defined by

$$\begin{aligned} |D| &= [s\beta_2 + s\phi\beta_1(1-c_1) + (1+\beta_1)] > 0 \\ \phi &= \frac{1}{(\eta + c_2)(1 - \frac{\lambda_1}{\lambda_2})} = \frac{1+\beta_1}{\eta + c_2(1+\eta_1)} \\ (cofD)^T &= \begin{pmatrix} s\beta_2 + (1+\beta_1) & -\phi(1-c_1)(1+\beta_1) & s\phi(1-c_1) \\ \beta_1 & -\beta_2 - \beta_1\phi(1-c_1) & -1 \\ -s\beta_1 & (1+\beta_1) & -s \end{pmatrix} \end{aligned}$$

The solution from equation (3.96) can be shown to be

$$\begin{aligned} &(a) \quad z_t = d_1 + \lambda_1 z_{t-1} + \xi_{1,t} \\ &(b) \quad y_t = d_2 + \xi_{2,t} \\ &(c) \quad e_t = d_3 + \frac{\lambda_1}{\lambda_2} z_{t-1} + \xi_{3,t} \\ &(d) \quad p_t = d_3 + \frac{\lambda_1}{\lambda_2} z_{t-1} + p_t^* + \xi_{3,t} \end{aligned} \tag{3.97}$$

where

$$d_1 = \frac{\phi g}{|D|}\{c_2(s\beta_2 + 1 + \beta_1)(1 - \frac{\lambda_1}{\lambda_2}) + (1-c_1)[(1+\beta_1)(1 - \frac{\lambda_1}{\lambda_2}) + s]\}$$

$$
\begin{aligned}
d_2 &= \frac{g}{|D|}\{\frac{s\beta_1 c_2}{\eta + c_2} + (1+\beta_1)(1-\frac{\lambda_1}{\lambda_2}) + s\} \\
d_3 &= \frac{g}{|D|}\{1 - \frac{\beta_1 c_2}{\eta + c_2} - [\beta_2 + \beta_1\phi(1-c_1)](1-\frac{\lambda_1}{\lambda_2})\} \\
\xi_{1,t} &= \frac{\phi}{|D|}\{-(1-c_1)(1+\beta_1)u_t + [s\beta_2 + (1+\beta_1)]w_t \\
&\quad -[c_2(1+\beta_1+s\beta_2) + s\beta_1(1-c_1)](p_t^* + r_t^*)\} \\
\xi_{2,t} &= \frac{1}{|D|}\{(1+\beta_1)u_t + s\beta_1\phi w_t + s\beta_1(1-\phi c_2)(p_t^* + r_t^*)\} \\
\xi_{3,t} &= \frac{1}{|D|}\{-[\beta_2 + \beta_1\phi(1-c_1)]u_t + \phi\beta_1 w_t - [\phi\beta_1 c_2 + s\beta_1\phi(1-c_1) \\
&\quad +(1+s\beta_2)]p_t^* - \beta_1(1-\phi c_2)r_t^*\}
\end{aligned}
$$

The asymptotic variances of y_t, z_t, e_t, and p_t are

$$
\begin{aligned}
&(a) \quad \sigma_y^2 = \sigma_{\xi_2}^2 \\
&(b) \quad \sigma_z^2 = \frac{\sigma_{\xi_1}^2}{1-\lambda_1{}^2} \\
&(c) \quad \sigma_e^2 = \frac{\lambda_1^2}{\lambda_2^2}\sigma_z^2 + \sigma_{\xi_3}^2 \\
&(d) \quad \sigma_p^2 = \frac{\lambda_1^2}{\lambda_2^2}\sigma_z^2 + \sigma_{\xi_3}^2 + \sigma_{p*}^2
\end{aligned}
\tag{3.98}
$$

By the assumption of the uncorrelated random disturbances, the asymptotic variance of the real income may be written as follows.

$$
\sigma_y^2 = \sigma_y^2(u) + \sigma_y^2(w) + \sigma_y^2(p^*) + \sigma_y^2(r^*) \tag{3.99}
$$

where

$$
\begin{aligned}
\sigma_y^2(u) &= \frac{(1+\beta_1)^2}{|D|^2}\sigma_u^2 \\
\sigma_y^2(w) &= \frac{(s\beta 1\phi)^2}{|D|^2}\sigma_w^2
\end{aligned}
$$

$$\sigma_y^2(p^*) = \frac{s^2\beta_1^2(1-\phi c_2)^2}{|D|^2}\sigma_{p^*}^2$$
$$\sigma_y^2(r^*) = \frac{s^2\beta_1{}^2(1-\phi c_2)^2}{|D|^2}\sigma_{r^*}^2$$

Similarly, the asymptotic variance of e_t can be written as follows.

$$\sigma_e^2 = \sigma_e^2(u) + \sigma_e^2(w) + \sigma_e^2(p^*) + \sigma_e^2(r^*) \tag{3.100}$$

where

$$\sigma_e^2(u) = \{\frac{\lambda_1^2}{\lambda_2^2}\frac{1}{1-\lambda_1^2}\frac{(1-c_1)^2(1+\beta_1)^2}{|D|^2} + \frac{[\beta_2 + \beta_2\phi(1-c_1)]^2}{|D|^2}\}\sigma_u^2$$
$$\sigma_e^2(w) = \{\frac{\lambda_1^2}{\lambda_2^2}\frac{1}{1-\lambda_1^2}\frac{[s\beta_2+(1+\beta_1)]^2}{|D|^2} + \frac{s^2\beta_1^2\phi^2}{|D|^2}\}\sigma_w^2$$
$$\sigma_e^2(p^*) = \{\frac{\lambda_1^2}{\lambda_2^2}\frac{1}{1-\lambda_1^2}\frac{[c_2(1+\beta_1+s\beta_2)+s\beta_1(1-c_1)]^2}{|D|^2} + \frac{[\phi\beta_1 c_2 + s\beta_1\phi(1-c_1)+(1+s\beta_2)]^2}{|D|^2}\}\sigma_{p^*}^2$$
$$\sigma_e^2(r^*) = \{\frac{\lambda_1^2}{\lambda_2^2}\frac{1}{1-\lambda_1^2}\frac{[c_2(1+\beta_1+s\beta_2)+s\beta_1(1-c_1)]^2}{|D|^2} + \frac{\beta_1^2(1-\phi c_2)^2}{|D|^2}\}\sigma_{r^*}^2$$

Finally, the asymptotic variance of price may also be written as follows.

$$\sigma_p^2 = \sigma_p^2(u) + \sigma_p^2(w) + \sigma_p^2(p^*) + \sigma_p^2(r^*) \tag{3.101}$$

where

$$\sigma_p^2(u) = \sigma_e^2(u)$$
$$\sigma_p^2(w) = \sigma_e^2(w)$$
$$\sigma_p^2(p^*) = \sigma_e^2(p^*) + \sigma_{p^*}^2$$
$$\sigma_p^2(r^*) = \sigma_e^2(r^*)$$

The income process under the fixed money supply rule is exogenous process as well. The process is determined by all the exogenous random disturbances of the system. But the price and exchange rate follow convergent time-dependent processes. Both processes depend on the previous exchange rate and interest rate, in addition to all the exogenous random disturbances. From the previous analysis, we have known that all the real output, price and exchange rate are exogenous processes under the feedback money supply rule with three target variables (y, p, e). According to the assumed criterion of smaller variance of real output to be achieved, our derived feedback money supply rule is at least not dominated by the fixed money supply.

The comparisons of the relative performance between the feedback and the fixed rule can be easily seen from the asymptotic variances of the real income, price and exchange rate in these two money supply rules. The more detailed and reliable comparisons of the relative magnitudes of each coefficient of the asymptotic variances in these two rules would be used to identify the variation of all the endogenous variables attributed to a variety of random shocks. However, to what extent of the insulation of feedback versus fixed money supply policy could be examined clearly from the time path of endogenous variables in the simulation experiments.

3.4 The Alternative Criterion - Price Forecast Error

It is known from aggregate supply function of equation (2.1) that aggregate output has response to the expectation error of price. The price forecast error may result in resource misallocation and welfare loss. For an economy with Lucas-type supply equation, a more appealing criteria, which has been suggested recently, is the minimization of the mean-square price forecast error of supplier. The objective function is

$$\min \sum_{t=1}^{\infty} \Pi^{t-1}(p_t - E_{t-1}p_t)^2, \quad 0 < \Pi < 1 \tag{3.102}$$

The meaning of equation (3.102) is equivalent to the minimization of output fluctuation when the information is perfect. The price forecast error is zero with full information, that is, $p_t - E_{t-1}p_t = 0$. It follows that the output with full information turns out to be $y_t^0 = u_t$, where y_t^0 is full information output level. The output supply is totally affected by the random supply innovation when the suppliers have full information in the price expectation and make no forecast error. Thus, the minimization of the mean square of actual output deviation from full information output is

$$\min \sum_{t=1}^{\infty} \Pi^{t-1} s^2 (p_t - E_{t-1}p_t)^2 \tag{3.103}$$

which is equivalent to equation (3.102).

Equations (3.2a)-(3.2d) can be reduced to state-space representation with one-state variable p_t, of the form

$$p_t = Ap_{t-1} + Bm_t + c(t) + \varepsilon(t) \tag{3.104}$$

where

$$\begin{aligned}
A &= \frac{1}{V}[\eta(1 + \frac{1}{\beta_1} + \frac{s\beta_2}{\beta_1})] \\
B &= \frac{1}{V}(\frac{\eta + c_2}{\beta_1}) \\
c(t) &= \frac{1}{V}\{s[(1 - c_1) + \frac{(\eta + c_2)\beta_2}{\beta_1} + \frac{\eta\beta_2}{\beta_1} - \frac{\eta}{s}]E_{t-1}p_t \\
&\quad - \frac{\eta\beta_2}{\beta_1}E_{t-2}p_{t-1} + (\eta + c_2)E_t p_{t+1} - \frac{\eta}{\beta_1}m_{t-1}\}
\end{aligned}$$

$$\varepsilon(t) = \frac{1}{V}\{-[(1-c_1)+(\frac{\eta+c_2}{\beta_1})\beta_2+\frac{\eta\beta_2}{\beta_1}]u_t+\frac{\eta\beta_2}{\beta_1}u_{t-1}$$
$$+\eta(r_t^*-r_{t-1}^*)+\frac{\eta}{s}(p_t^*-p_{t-1}^*)+w_t\}$$
$$V = (\eta+c_2)(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})+s[(1-c_1)+\frac{\eta\beta_2}{\beta_1}]$$

The optimal control problem is to choose m_t so as to minimize equation (3.102), subject to equations (3.104), (3.13) and (3.14). From equation (3.27), the optimal money supply is easily derived as

$$m_t = \frac{-\eta(1+\frac{1}{\beta_1}+\frac{s\beta_2}{\beta_1})}{(\frac{\eta+c_2}{\beta_1})}p_{t-1}+\frac{\eta}{(\eta+c_2)}m_{t-1}+\frac{\eta\beta_2}{(\eta+c_2)}E_{t-2}p_{t-1}$$
$$+\frac{1}{(\frac{\eta+c_2}{\beta_1})}[\eta+(\eta+c_2)(1+\frac{1}{\beta_1})]E_{t-1}p_t-\beta_1E_tp_{t+1} \quad (3.105)$$

In addition to lagged money supply (m_{t-1}) and the previous expectation price ($E_{t-2}p_{t-1}$ and $E_{t-1}p_t$) and current expectation price (E_tp_{t+1}), the derived feedback money supply rule only depends on lagged target variable (p_{t-1}). Equation (3.105) indicates that the current money supply policy is negatively responsive to previous price and current expected price but positively responsive to previous money supply and previous expected price.

Substituting the derived feedback money supply rule of equation (3.105) into equation (3.104), we have the final form of price p_t,

$$p_t = E_{t-1}p_t-\frac{1}{v}\{[(1-c_1)+(\frac{\eta+c_2}{\beta_1})\beta_2+\frac{\eta\beta_2}{\beta_1}]u_t+\frac{\eta\beta_2}{\beta_1}u_{t-1}$$
$$+\eta(r_t^*-r_{t-1}^*)+\frac{\eta}{s}(p_t^*-p_{t-1}^*)+w_t\} \quad (3.106)$$

The expectational error of price is

$$p_t-E_{t-1}p_t = -\frac{1}{V}\{[(1-c_1)+(\frac{\eta+c_2}{\beta_1})\beta_2+\frac{\eta\beta_2}{\beta_1}]u_t+\frac{\eta\beta_2}{\beta_1}u_{t-1}$$

$$+\eta(r_t^* - r_{t-1}^* + \frac{\eta}{s}(p_t^* - p_{t-1}^*) + w_t\} \qquad (3.107)$$

The price forecast error $(p_t - E_{t-1}p_t)$ is negatively related to all the current random disturbances. It implies that the unanticipated component of price will tend to be smaller in response to all the current random disturbances.

We shall compare how the magnitude of price forecast error is related to a variety of random disturbances in the three target variables case in section 3.1 and the case of minimization of price forecast error. From equation (3.40), the price forecast error in the three target variable case is

$$p_t - E_{t-1}p_t = \frac{1}{|D|}[-\phi u_t + \eta p_t^* + \eta(r_t^* - r_{t-1}^*) + w_t] \qquad (3.108)$$

The price forecast error turns out smaller in response to the current supply innovations but larger in response to other random disturbances.

4 MICROECONOMIC FOUNDATIONS OF DEMAND SIDE

As we have seen before that the demand function for consumption and money in T-B model are built-up in ad hoc fashions. Theoretically, the consumption and money demand functions must be simultaneously determined, rather than independently built-up, from the consumer maximization problem. From this standpoint of view, the demand function for consumption and money in T-B model appear more weak and shaky in aspect of the theoretical interpretation of consumer behavior. Sofaras, there is no rationale based on the insights from the consumer behavior to support the setup of these two functions in equations (2.2) and (2.7).

In this chapter, the main work is devoted to explore how the consumption and money demand are determined simultaneously from the microeconomic foundations of utility maximization problem of the consumer behavior. More specifically, our attempt is to examine the nature of both functions and the explanatory factors in the consumption and money demand decisions with the dynamic framework of utility maximization of the consumer.

We shall proceed to analyze theoretically our concerned issue from two distinct theoretical models: the general equilibrium cash-in-advance monetary model, and overlapping generations model. These two kinds of monetary models are built up from the assumptions about the behavior of the individual agent.

4.1 The Cash-in-Advance Model

The cash-in-advance monetary model is now developed with the restriction of exchange pattern that requires goods be purchased with money accumulated in advance. The central feature of this model is the hinge on the transaction motivation of money demand for the purchase of the good. The main purpose of the setup is to derive demand for consumption and money from the consumer optimization behavior with the money holdings motivated by the cash-in-advance constraint.

The assumptions about tastes, technology, institutional arrangements, and the behavior of economic units such as the consumer and the government are made for the sake of the analysis simplification. However, the assumptions have no loss of generality of the implications of the comparative results.

The consumers are infinite-lived. The preference of the consumer is identical. The country produces one nonstorable consumption goods. The goods is simply in the form of endowment accrued to the consumer each period. The endowment each period is serially uncorrelated random variable. The consumer's optimal allocation decisions on consumption and portfolio are derived from the utility maximization behavior over his infinite lifetime.

The money is held because the transaction mechanism dictates that the currency be used to purchase goods. Specifically, money buys goods but goods cannot buy money. The cash-in-advance constraint is imposed to the trader to the effect that goods transaction must be financed by the money accumulated in advance[1].

[1]The version of cash-in-advance constraint originated from Clower (1967) and Tsiang (1956). There are many well-developed one-country and two-country versions of cash-in-advance model to examine the determination of equilibrium commodity prices, asset prices and exchange rate (Stockman (1980), Lucas (1982),

There are two trading sessions each period: commodity and asset trading sessions. These two trading sessions open alternately. The information about the current state of the economy is known to all the agents at the beginning of each period. The timing of information is crucial in the aspects of the equilibrium solutions of the model. There are totally two kinds of timing of the events in the literature. The timing of the events in Lucas' (1982) paper is different from that in Svensson (1985) and Stockman (1980). In Lucas' model, the asset trading session is open first within each period, and then the goods trading session begins after the asset trading session is over. It is just opposite to Svensson and Stockman.

We will follow the timing of information of Svensson and Stockman ending up with demand function for money with variable velocity instead of unitary velocity. One-country and two-country model will be discussed in the following sections.

4.1.1 One-country model

4.1.1.1 The optimization problem of the consumer Each consumer preference is identical over the consumption stream. The representative consumer chooses consumption and money (or portfolio allocations) to maximize the discounted infinite sum of present and future utilities

$$E[\sum_{t=1}^{\infty} \beta^{t-1} U(C_t)] \tag{4.1}$$

where $0 < \beta < 1$, β is discounted factor, C_t is the consumption in period t. $U(\bullet)$ is current period utility function which is continuously differentiable, bounded, increasing and strictly concave. The infinite lifetime utility function is additively

Svensson (1985a and 1985b), Stockman and Svensson (1987)).

separable with respect to time. The current utility function is a function of consumption goods. $E[\bullet]$ is a conditional expectation operator.

At the start of each period, the consumer carries the currency holdings which are accumulated from last period and then enters goods market to purchase consumption goods. After the goods transaction is completed, he may carry unspent money to the end of the period. At the end of each period, he receives his endowment paid in cash and monetary transfer payment from the government.

At the beginning of period t, the representative consumer purchases consumption goods in accord with the cash-in-advance constraint

$$C_t \leq \frac{M_{t-1}}{P_t} \tag{4.2}$$

where P_t is the price of the goods. M_{t-1} is the currency carried from $t-1$ to t. The cash-in-advance constraint is not necessary to be binding for the existence of positive interest rate[2].

At the end of period t when he makes decision on money holdings, measured in the units of the goods, the consumer is facing the budget constraint

$$\frac{M_t}{P_t} = y_t + \tau_t + (\frac{M_{t-1}}{P_t} - C_t) \tag{4.3}$$

where y_t is the endowment in period t. τ_t is the real monetary transfer payment (net taxes), measured in the units of the goods. The right hand side of equation (4.3) is real income received in cash, real monetary transfer payment, and unspent

[2]Nonbinding cash-in-advance constraint is allowed to get a more general and reasonable demand for money. The positive nominal interest rates are not longer inconsistent with a nonbinding cash-in-advance constraint in some states (See Svensson (1985a)).

money. Equation (4.3) simply expresses that the money holdings decision at t is subject to the total real wealth available at the end of that period.

The budget constraint of equation (4.3) is actually the standard familiar budget constraint. It can be rearranged to become the standard form:

$$C_t + \frac{M_t}{P_t} = y_t + \tau_t + \frac{M_{t-1}}{P_t} \equiv \Theta_t \tag{4.4}$$

The right hand side of equation (4.4) is total wealth available to the representative domestic consumer in period t (Θ_t). The wealth evolves according to

$$\Theta_{t+1} = y_{t+1} + \tau_{t+1} + \frac{M_t}{P_{t+1}} \tag{4.5}$$

Assume that government issues money and is transferred to the public via the monetary transfer payment and reduces money from tax collection. The government monetary transfer payment is simply new creation of money, in accord with the equation

$$\tau_t = \frac{\overline{M}_t}{P_t} - \frac{\overline{M}_{t-1}}{P_t} \tag{4.6}$$

where $\overline{M}_t$ is the money supply at the end of period t.

Let $m_t \equiv \frac{M_t}{P_t}$, m_t is real balance held by consumer from t to $t+1$, in terms of goods price at period t. Thus, combining equation (4.6), the budget constraint of equation (4.4) and cash-in-advance constraint of equation (4.2) can be rewritten as

$$\begin{aligned} &(a) \quad C_t + m_t = y_t + m_{t-1}\frac{P_{t-1}}{P_t} + (\overline{m}_t - \overline{m}_{t-1}\frac{P_{t-1}}{P_t}) \\ &(b) \quad C_t \le m_{t-1}\frac{P_{t-1}}{P_t} \end{aligned} \tag{4.7}$$

The consumer's dynamic optimization problem can now be represented as follows. The consumer chooses decision policies for $\{C_t, m_t\}_{t=1}^{\infty}$ to maximize

$$E[\sum_{t=1}^{\infty} \beta^{t-1} U(C_t)] \qquad \text{with } M_0 \text{ given} \tag{4.8}$$

subject to

$$\begin{aligned}
C_t + m_t &= y_t + m_{t-1}\frac{P_{t-1}}{P_t} + (\overline{m}_t - \overline{m}_{t-1}\frac{P_{t-1}}{P_t}) \\
C_t &\leq m_{t-1}\frac{P_{t-1}}{P_t} \\
\Theta_{t+1} &= y_{t+1} + m_t\frac{P_t}{P_{t+1}} + (\overline{m}_{t+1} - \overline{m}_t\frac{P_{t-1}}{P_t})
\end{aligned}$$

With the property that the objective function is additively separable with respect to time and the decision set is convex and compact, the dynamic programming is applicable to solve this dynamic maximization problem. The state variables at time t are Θ_t, m_{t-1} and $\overline{m}_t$. The choice variables are C_t ands m_t. The value function of the objective function is written as

$$V(\Theta_t, m_{t-1}, \overline{m}_t) = \max\{U(C_t) + \beta E[V(\Theta_{t+1}, m_t, \overline{m}_{t+1})]\} \tag{4.9}$$

This is Bellman's equation for the dynamic optimization problem.

To solve the constrained-maximization problem on the right hand side of the value functional equation, we form the Lagrangian,

$$\begin{aligned}
\mathcal{L} &= \{U(C_t) + \beta E[V(\Theta_{t+1}, m_t, \overline{m}_{t+1})]\} \\
&\quad + \lambda_t[y_t + m_{t-1}\frac{P_{t-1}}{P_t} + (\overline{m}_t - \overline{m}_{t-1}\frac{P_{t-1}}{P_t}) - C_t - m_t] \\
&\quad + \psi_t[m_{t-1}\frac{P_{t-1}}{P_t} - C_t]
\end{aligned} \tag{4.10}$$

where λ_t, ψ_t are Lagrangian multipliers associated with the budget constraint and the cash-in-advance constraint respectively. Taking the first derivative with respect to the decision variables, C_t and m_t, and Lagrangian multipliers, λ_t and ψ_t, the necessary conditions for the solutions of the utility maximization are:

$$(a) \quad \frac{\partial U}{\partial C_t} = \lambda_t + \psi_t$$

$$(b)\quad \beta E[\frac{P_t}{P_{t+1}}(\lambda_{t+1} + \psi_{t+1})] = \lambda_t$$

$$(c)\quad m_{t-1}\frac{P_{t-1}}{P_t} - C_t \geq 0, \qquad \psi_t \geq 0, \qquad (m_{t-1}\frac{P_{t-1}}{P_t} - C_t)\psi_t = 0 \tag{4.11}$$

By Benveniste and Scheinkman formula, we have that

$$\frac{\partial V}{\partial m_{t-1}} = \frac{P_{t-1}}{P_t}(\lambda_t + \psi_t) \tag{4.12}$$

which has been used in the above first-order conditions.

The Lagrangian multiplier λ is the marginal utility of real wealth, measured in the units of the goods. ψ is the marginal utility of real money balance, measured in the units of the goods.

Equation (4.11a) tells us that the marginal utility of goods consumption is equal to the sum of marginal utility of real wealth and real balances. If the cash-in-advance constraint is nonbinding, it follows that money provides no liquidity service and serves essentially as a store-of-value. In this case, the marginal utility of goods consumption is exactly equal to the marginal utility of real wealth, analogous to the nonmonetary equilibrium condition in an economy without money. Svensson (1985a) gives an attractive explanation about the services that the currency has provided: " the existence of a binding cash-in-advance constraint drives a wedge between the marginal utility of wealth and that of consumption, since wealth cannot instantaneously be used to purchase consumption".

Equation (4.11b) can be rewritten as

$$\beta E[\frac{1}{P_t}(\lambda_{t+1} + \psi_{t+1})] = \frac{\lambda_t}{P_t} \tag{4.13}$$

This is the necessary condition for the solution of money demand that maximizes the consumer lifetime expected utility. The bracket term on the left hand side of equation (4.13) is the marginal utility of real wealth including the liquidity service of money deflated by the goods price, that is, the marginal utility obtained from one-dollar wealth. Equation (4.13) expresses that the consumer holds the quantity of money up to the point where the discounted expected marginal utility of one dollar future wealth exactly equates the marginal utility of one dollar current wealth.

The marginal utility of next period consumption equates the sum of marginal utility of next period real wealth and real balances

$$\frac{\partial U}{\partial C_{t+1}} = \lambda_{t+1} + \psi_{t+1} \tag{4.14}$$

which follows from the equation (4.11a). Then substituting equation (4.14) into equation (4.13), we have that

$$\beta E[\frac{1}{P_{t+1}}(\frac{\partial U}{\partial C_{t+1}})] = \frac{1}{P_t}\frac{\partial U}{\partial C_t} - \psi_t \tag{4.15}$$

Equation (4.15) demonstrates that the marginal rate of substitution between current and future consumption is no longer exactly equal to the intertemporal price ratio. This distinct result from the one in a model without money lies in the fact that the liquidity service of money has been taken into account in our model.

If the cash-in-advance constraint is nonbinding, *i.e.*, $\psi = 0$, then equation (4.15) becomes our familiar intertemporal equilibrium condition between current and future consumption.

$$\beta E[\frac{1}{P_{t+1}}(\frac{\partial U}{\partial C_{t+1}})] = \frac{\frac{\partial U}{\partial C_t}}{P_t} \tag{4.16}$$

If the economy is certainty, it turns out to be the formula $\frac{\partial U}{\partial C_{t+1}} / \frac{\partial U}{\partial C_t} = \frac{P_{t+1}}{\beta P_t}$. Equation (4.15) is in deed the arbitrage condition for consumption and money demand. The current money holdings carried to next period would be realized its utility by virtue of the next period consumption.

4.1.1.2 The solution of the optimization problem Now, we are in attempt to get at the explicit equilibrium solution of consumption and money demand. The functional form of consumer's utility function is needed to be specified in order to get at explicit solutions. The current utility function is assumed to be

$$U_t = \frac{C^{1-\alpha}}{1-\alpha} \qquad 0 < \alpha < 1 \tag{4.17}$$

where α is the measure of the relative risk aversion.

The necessary conditions for consumer optimization problem from equations (4.11a)-(4.11c) are

$$\begin{aligned}
&(a) \qquad C_t^{-\alpha} = \lambda_t + \psi_t \\
&(b) \qquad \beta E[\frac{P_t}{P_{t+1}} C_{t+1}^{-\alpha}] = \lambda_t \\
&(c) \quad m_{t-1}\frac{P_{t-1}}{P_t} - C_t \geq 0, \qquad \psi_t \geq 0, \qquad (m_{t-1}\frac{P_{t-1}}{P_t} - C_t)\psi_t = 0
\end{aligned} \tag{4.18}$$

Firs, consider the case that cash-in-advance constraint is always binding. It follows from equation (4.18c) that $C_t = m_{t-1}\frac{P_{t-1}}{P_t}$. The consumer spends all money he has. In this case, the endowment and monetary transfer payment he receives are totally converted to be real money balance for the next period consumption, $m_t = y_t + (z_t - 1)m_{t-1}\frac{P_{t-1}}{P_t}$, where z_t is the gross growth rate of money supply at time t. The sequence of $\{C_t, m_t\}$ must satisfy the feasibility, utility maximization, and market clearings.

Definition of equilibrium: an equilibrium is the sequence $\{C_t, m_t\}_{t=1}^{\infty}$ such that

$$(i)\, V(\Theta_t, m_{t-1}, \overline{m}_t) = \max_{c,m}\{U(C_t) + \beta E[V(\Theta_{t+1}, m_t, \overline{m}_{t+1})]\}$$

subject to

$$\begin{aligned} C_t + m_t &= y_t + m_{t-1}\frac{P_{t-1}}{P_t} + (\overline{m}_t - \overline{m}_{t-1}\frac{P_{t-1}}{P_t}) \\ C_t &\leq m_{t-1}\frac{P_{t-1}}{P_t} \end{aligned} \tag{4.19}$$

(ii) For each vector $(y_t, P_t, \overline{m}_t), V(\Theta_t, m_{t-1}, \overline{m}_t)$

satisfies the following market equilibrium conditions

$$(a) y_t = C_t$$

$$(b) m_t = \overline{m}_t$$

Next, consider the case that the cash-in-advance constraint is always nonbinding, $\psi_t = 0$ for all t. From the necessary conditions (4.18a) and (4.18b), we have the first-order difference equation of consumption

$$C_t = [\beta E(\frac{P_t}{P_{t+1}} C_{t+1}^{-\alpha})]^{\frac{-1}{\alpha}} \tag{4.20}$$

The consumption solution can be solved from equation (4.20) as a function of the time discounted factor, probability distribution of real income, risk aversion altitudes and expected inflation rate. Once the equilibrium solution of consumption is determined by the equation (4.20), then the substitution of equation (4.20) into the standard budget constraint, the money demand is obtained as a function of current endowment, growth rate of money supply, current and expected inflation rate, time discounted factor and risk aversion altitudes.

4.1.2 Two-country model

We shall extend one-country model in the previous subsection into two-country model. The model is an attempt to examine the insights which are derived from the optimization problem of the consumer in both countries into the nature of consumption and money demand functions.

The assumption about tastes and the behavior of economic agents such as consumer and government is symmetric to both countries. The preference of the consumer is identical across countries. The countries have identical constant populations or the same structure of the population. Each country produces one nonstorable and nonreproducible consumption goods. Domestic country produces good one and foreign country produces goods two. The output in both countries is random variable, having a certain probability distribution.

The two currencies are held because the transaction mechanism dictates that domestic currency be used to purchase domestic goods and foreign currency be used to purchase foreign goods. That is, money holdings are motivated by cash-in-advance constraints and the convention or legal restriction is adopted that goods must be paid for by using the seller's currency.

The lump-sum monetary transfer payment (or net taxes) is effected to the domestic consumer at the end of each period.

4.1.2.1 The optimization problem of the consumer The representative consumer in domestic country chooses consumption and portfolio allocations on domestic and foreign currencies to maximize the discounted infinite sum of present and future utilities

$$E\{\sum_{t=1}^{\infty} \beta^{t-1} U(C_{1t}^{1}, C_{2t}^{1})\} \tag{4.21}$$

where $0 < \beta < 1$, β is discounted factor which is identical across countries. C_{it}^{j} is the goods i consumed by the consumer in country j in period t, $i, j = 1, 2$ (1 is denoted as domestic country and 2 is denoted as foreign country). The current utility function is a function of current consumption of domestic and foreign goods.

Analogously, the representative consumer in foreign country chooses consumption and portfolio allocations of domestic and foreign currencies to maximize his discounted infinite sum of present and future utilities

$$E\{\sum_{t=1}^{\infty} \beta^{t-1} U(C_{1t}^{2}, C_{2t}^{2})\}, \qquad 0 < \beta < 1 \tag{4.22}$$

where C_{1t}^{2} and C_{2t}^{2} are the domestic and foreign goods consumed by foreign consumer in period t respectively.

At the start of period t the domestic consumer is facing the cash-in-advance constraints when he enters the goods markets

$$\begin{aligned} (a) \quad & C_{1t}^{1} \leq \frac{M_{1,t-1}^{1}}{P_{1t}} \\ (b) \quad & C_{2t}^{1} \leq \frac{M_{2,t-1}^{1}}{P_{2t}} \end{aligned} \tag{4.23}$$

where P_{1t} and P_{2t} are the own currency price of domestic goods 1 and foreign goods 2, respectively. M_{it}^{j} is the country i currency held by the consumer in country j in period t. According to the transaction convention that domestic currency is used to pay for domestic goods and foreign currency is used to pay for foreign goods, the domestic goods transaction is constrained by equation (4.23a) and the foreign goods is constrained by equation (4.23b). These two cash-in-advance constraints are not necessary to be binding for the existence of positive interest rate.

After goods transaction is completed, the consumer enters the asset market possibly with unspent currencies. At the end of each period, he will receive his current endowment paid in cash and monetary transfer payment (or net tax) from domestic government, and then rearranges his portfolio allocations on domestic and foreign currencies which are accumulated for the next period purchases of domestic and foreign consumption goods. The consumer is facing the budget constraint during the asset trading session, measured in the units of goods 1

$$\frac{M_{1t}^1}{P_{1t}} + \frac{e_t M_{2t}^1}{P_{1t}} = y_{1t} + \tau_{1t} + (\frac{M_{1,t-1}^1}{P_{1t}} - C_{1t}^1) + \frac{e_t}{P_{1t}}(M_{2,t-1}^1 - P_{2t}C_{2t}^1) \quad (4.24)$$

where e_t is the exchange rate at period t. y_{1t} is the endowment of domestic consumer at period t. The right hand side of equation (4.24) is the real current endowment paid in cash (y_{1t}), the real monetary transfer payment (τ_{1t}), unspent domestic currency $(\frac{M_{1,t-1}^1}{P_{1t}} - C_{1t}^1)$, and unspent foreign currency, $(\frac{e_t}{P_{1t}}(M_{2,t-1}^1 - P_{2t}C_{2t}^1))$, measured in the units of the goods 1. The left hand side of equation (4.24) expresses the portfolio allocations on domestic and foreign currencies.

The budget constraint of equation (4.24) can be rearranged to become the standard form:

$$C_{1t}^1 + \frac{e_t P_{2t}}{P_{1t}} C_{2t}^1 + \frac{M_{1t}^1}{P_{1t}} + \frac{e_t M_{2t}^1}{P_{1t}} = y_{1t} + \tau_{1t} + \frac{M_{1,t-1}^1}{P_{1t}} + \frac{e_t M_{2,t-1}^1}{P_{1t}} \equiv \Theta_{1t} \quad (4.25)$$

where Θ_{1t} is total wealth available to domestic consumer in period t. The asset transactions are constrained by equation (4.25), given equations (4.23a) and (4.23b).

The total wealth evolves according to

$$\Theta_{1,t+1} = y_{1,t+1} + \tau_{1,t+1} + \frac{M_{1t}}{P_{1,t+1}} + \frac{e_{t+1}M_{2t}}{P_{2,t+1}} \tag{4.26}$$

The real monetary transfer payment in domestic and foreign countries are according to

$$\tau_{1t} = \frac{\overline{M}_{1t}}{P_{1t}} - \frac{\overline{M}_{1,t+1}}{P_{1t}} \tag{4.27}$$

$$\tau_{2t} = \frac{\overline{M}_{2t}}{P_{2t}} - \frac{\overline{M}_{2,t+1}}{P_{2t}} \tag{4.28}$$

where $\overline{M}_{1t}$ and $\overline{M}_{2t}$ are the money supplies of domestic and foreign countries at period t. τ_{1t} and τ_{2t} are the real monetary transfer payments of domestic and foreign countries in terms of goods 1 and 2, respectively.

Let $\overline{m}_{1t} = \frac{\overline{M}_{1t}}{P_{1t}}$ and $\overline{m}_{2t} = \frac{M_{2t}}{P_{2t}}$, $\overline{m}_{1t}$ and $\overline{m}_{2t}$ are domestic and foreign real money supply at time t, respectively. Let $m^1_{1t} = \frac{M^1_{1t}}{P_{1t}}$ and $m^1_{2t} = \frac{M^1_{2t}}{P_{2t}}$, m^1_{1t} and m^1_{2t} are domestic and foreign real balances held by domestic consumer at time t, respectively. Combining equation (4.27), budget constraint and the cash-in-advance constraints, the domestic representative consumer's dynamic optimization problem can be represented as follows. The domestic consumer chooses the decision policies for $\{C^1_{1t}, C^1_{2t}, m^1_{1t}, m^1_{2t}\}^{\infty}_{t=1}$ so as to maximize

$$E\{\sum_{t=1}^{\infty} \beta^{t-1} U(C^1_{1t}, C^1_{2t})\}$$

subject to

$$\begin{aligned} & C^1_{1t} + \frac{e_t P_{2t}}{P_{1t}} C^1_{2t} + m^1_{1t} + e_t m^1_{2t} \frac{P_{2t}}{P_{1t}} \\ = \; & y_{1t} + (\overline{m}_{1t} - \overline{m}_{t-1} \frac{P_{1,t-1}}{P_{1t}}) + m^1_{1,t-1} \frac{P_{1,t-1}}{P_{1t}} + e_t m^1_{2,t-1} \frac{P_{2,t-1}}{P_{1t}} \end{aligned}$$

$$
\begin{aligned}
&\equiv \Theta_{1t} \\
C_{1t}^1 &\leq m_{1,t-1}^1 \frac{P_{1,t-1}}{P_{1t}} \\
C_{2t}^1 &\leq m_{2,t-1}^1 \frac{P_{2,t-1}}{P_{2t}} \\
\Theta_{1,t+1} &= y_{t+1} + (\overline{m}_{1,t+1} - \overline{m}_{1t} \frac{P_{1t}}{P_{1,t+1}}) + m_{1t}^1 \frac{P_{1t}}{P_{1,t+1}} \\
&+ e_{t+1} m_{2t}^1 \frac{P_{2t}}{P_{1,t+1}} \qquad (4.29)
\end{aligned}
$$

For this dynamic optimization problem, the state variables at time t are y_{1t}, $m_{1,t-1}^1$, and $\overline{m}_{1t}$. The choice variables are C_{1t}^1, C_{2t}^1, m_{1t}^1, m_{2t}^1. The value function of the objective function is written as

$$
V(\Theta_{1t}, M_{1,t-1}^1, M_{2,t-1}^1) = \max\{U(C_{1t}^1, C_{2t}^1) + \beta E[V(\Theta_{1,t+1}, M_{1t}^1, M_{2t}^1)]\} \qquad (4.30)
$$

To solve the constrained-maximization problem on the right hand side of the value functional equation, we form the Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \{U(C_{1t}^1, C_{2t}^1) + \beta E[V(\Theta_{t+1}, m_{1t}^1, m_{2t}^1)]\} \\
&+\lambda_t[y_{1t} + (\overline{m}_{1t} - \overline{m}_{1,t-1} \frac{P_{1,t-1}}{P_{1t}}) + m_{1,t-1}^1 \frac{P_{1,t-1}}{P_{1t}} + e_t m_{2,t-1}^1 \frac{P_{2,t-1}}{P_{1t}} \\
&- C_{1t}^1 - \frac{e_t P_{2t}}{P_{1t}} C_{2t}^1 - m_{1t}^1 - e_t m_{2t}^1 \frac{P_{2t}}{P1t}] + \psi_t(m_{1,t-1}^1 \frac{P_{1,t-1}}{P_{1t}} - C_{1t}^1) \\
&+\nu_t(m_{2,t-1}^1 \frac{P_{2,t-1}}{P_{2t}} - C_{2t}^1) \qquad (4.31)
\end{aligned}
$$

where λ_t, ψ_t, and ν_t are Lagrangian multipliers associated with the budget constraint and the cash-in-advance constraints respectively. Taking the first derivative with respect to the decision variables, $C_{1t}^1, C_{2t}^1, m_{1t}^1, m_{2t}^1$, and the Lagrangian multipliers, λ_t, ψ_t, ν_t, the necessary conditions for the solutions of the utility maximization are:

$$
\begin{aligned}
&(a) \quad \frac{\partial U}{\partial C^1_{1t}} = \lambda_t + \psi_t \\
&(b) \quad \frac{\partial U}{\partial C^1_{2t}} = \lambda_t \frac{e_t P_{2t}}{P_{1t}} + \nu_t \\
&(c) \quad \beta E[\frac{1}{P_{1,t+1}}(\lambda_{t+1} + \psi_{t+1})] = \frac{\lambda_t}{P_{1t}} \\
&(d) \quad \beta E[\frac{P_{2t}}{P_{2,t+1}}(\frac{e_{t+1}P_{2,t+1}}{P_{1,t+1}}\lambda_{t+1} + \nu_{t+1})] = \lambda_t \frac{e_t P_{2t}}{P_{1t}} \\
&(f) \quad m^1_{1,t-1}\frac{P_{1,t-1}}{P_{1t}} - C^1_{1t} \geq 0,\ \psi_t \geq 0,\ (m^1_{1,t-1}\frac{P_{1,t-1}}{P_{1t}} - C^1_{1t})\psi_t = 0 \\
&(g) \quad m^1_{2,t-1}\frac{P_{2,t-1}}{P_{2t}} - C^1_{2t} \geq 0,\ \nu_t \geq 0,\ (m^1_{2,t-1}\frac{P_{2,t-1}}{P_{2t}} - C^1_{2t})\nu_t = 0
\end{aligned}
\tag{4.32}
$$

By Benveniste and Scheinkman formula, we have that

$$
\begin{aligned}
&(a) \quad \frac{\partial V}{\partial m^1_{1,t-1}} = \frac{P_{1,t-1}}{P_{1t}}(\lambda_t + \psi_t) \\
&(b) \quad \frac{\partial V}{\partial m^1_{2,t-1}} = \frac{e_t P_{2,t-1}}{P_{1t}}\lambda_t + \frac{P_{2,t-1}}{P_{2t}}\nu_t
\end{aligned}
\tag{4.33}
$$

which have been used in the above first-order conditions.

The optimization problem of the representative consumer in the foreign country is to choose the decision variables, C^2_{1t}, C^2_{2t}, m^2_{1t}, m^2_{2t} that maximize

$$
E[\sum_{t=1}^{\infty} \beta^{t-1} U(C^2_{1t}, C^2_{2t})], \qquad 0 < \beta < 1 \tag{4.34}
$$

subject to the budget constraint and cash-in-advance constraints, measured in the units of the domestic goods 1

$$
\begin{aligned}
& C^2_{1t} + \frac{e_t P_{2t}}{P_{1t}} C^2_{2t} + m^2_{1t} + e_t m^2_{2t} \frac{P_{2t}}{P_{1t}} \\
= \ & \frac{P_{2t}}{P_{1t}} y_{2t} + \frac{e_t}{P_{1t}}(\overline{m}_{2t} P_{2t}
\end{aligned}
$$

$$
\begin{aligned}
& -\overline{m}_{2,t-1}P_{2,t-1}) + m^2_{1,t-1}\frac{P_{1,t-1}}{P_{1t}} + e_t m^2_{2,t-1}\frac{P_{2,t-1}}{p_{1t}} \\
\equiv\ & \Theta_{2t} \\
C^2_{1t} \ \leq\ & m^2_{1,t-1}\frac{P_{1,t-1}}{P_{1t}} \\
C^2_{2t} \ \leq\ & m^2_{2,t-1}\frac{P_{2,t-1}}{P_{2t}} \\
\Theta_{2,t+1} \ =\ & \frac{P_{2,t+1}}{P_{1,t+1}}y_{2,t+1} + \frac{e_{t+1}}{P_{1,t+1}}(\overline{m}_{2,t+1}P_{2,t+1} - \overline{m}_{2t}P_{2t} \\
& + m1t^2\frac{P_{1t}}{P_{1,t+1}} + e_{t+1}m^2_{2t}\frac{P_{2t}}{P_{1,t+1}} \qquad (4.35)
\end{aligned}
$$

For simplicity, the time discount factor β is assumed to be identical across countries. The solutions for the optimization problem of the foreign representative consumer.

The Lagrangian multiplier λ is the marginal utility of real wealth measured in the units of domestic goods. ψ is the marginal utility of real domestic money measured in the units of domestic goods. ν is the marginal utility of real foreign money measured in the units of foreign goods.

Equation (4.32a) is the necessary condition for the equilibrium domestic consumption of domestic goods that the marginal utility of domestic goods consumption is equal to the sum of marginal utility of real wealth and real domestic money. Equation (4.32b) is the necessary condition for the equilibrium domestic consumption of foreign goods that the marginal utility of foreign goods consumption is equal to marginal utility of real wealth measured in the units of foreign goods plus marginal utility of real foreign money. If the two cash-in-advance constraints are nonbinding, *i.e.*, $\psi_t = 0$ and $\nu_t = 0$ followed from equations (4.32f) and (4.32g), then the marginal rate of substitution between domestic and foreign goods consumption is exactly equal to the domestic and foreign goods price ratio,

$$\frac{\partial U}{\partial C^1_{1t}} / \frac{\partial U}{\partial C^1_{2t}} = \frac{P_{1t}}{e_t P_{2t}}$$

This formula of marginal rate of substitution is the familiar one in the consumer model without money.

Equations (4.32a) and (4.32c) express that the domestic consumer holds the quantity of domestic money up to the point where the discounted expected marginal utility of one unit of domestic currency spent on future domestic goods consumption is exactly equal to the marginal utility of one unit of domestic money wealth. Equations (4.32b) and (4.32d) states that the domestic consumer holds the quantity of foreign currency up to the point where the discounted expected marginal utility of one unit of foreign currency spent on future foreign goods consumption is exactly equal to the marginal utility of one unit of foreign money wealth.

From equations (4.32c) and (4.32d), we have the equilibrium condition for the domestic and foreign money holdings that

$$E[\frac{1}{P_{2,t+1}}(\frac{\partial U}{\partial C^1_{2,t+1}})] = e_t E_t[\frac{1}{P_{1,t+1}}(\frac{\partial U}{\partial C^1_{1,t+1}})] \tag{4.36}$$

The allocation on domestic and foreign real money reaches the level where the expected marginal utility of one unit of domestic currency spent on the next period consumption of domestic goods equates the expected marginal utility of one unit of foreign currency spent on the next period consumption of foreign goods in terms of domestic currency

If both cash-in-advance constraints are binding, it follows from equations (4.32f) and (4.32g) that $\psi_t \neq 0$, $\nu_t \neq 0$, that is, the domestic and foreign money provide the liquidity services. In this case, both currencies serve essentially as medium of exchange. The domestic and foreign goods consumptions are

$$
\begin{aligned}
&(a)\quad C^1_{1t} = m^1_{1,t-1}\frac{P_{1,t-1}}{P_{1t}} \\
&(b)\quad C^1_{2t} = m^1_{2,t-1}\frac{P_{2,t-1}}{P_{2t}}
\end{aligned} \tag{4.37}
$$

The demand for domestic and foreign currency, m^1_{1t} and m^1_{2t} must satisfy the following equations simultaneously

$$
\begin{aligned}
&(a)\quad m^1_{1t} + e_t m^1_{2t}\frac{P_{2,t1}}{P_{1t}} = y_t + (\overline{m}_{1t} - \overline{m}_{1,t-1}\frac{P_{1,t-1}}{P_{1t}}) \\
&(b)\quad E[\frac{1}{P_{2,t+1}}(\frac{\partial U}{\partial C^1_{2,t+1}})] = e_t E[\frac{1}{P_{1,t+1}}(\frac{\partial U}{\partial C^1_{1,t+1}})]
\end{aligned} \tag{4.38}
$$

If both cash-in-advance constraints are always nonbinding, the value of $\psi_t = 0$ and $\nu_t = 0$ for all t. The necessary condition in equation (4.32) comes out to be

$$
\begin{aligned}
&(a)\quad \frac{\partial U}{\partial C^1_{1t}} = \lambda_t \\
&(b)\quad \frac{\partial U}{\partial C^1_{2t}} = \lambda_t\frac{e_t P_{2t}}{P_{1t}} \\
&(c)\quad \beta E(\frac{1}{P_{1,t+1}}\lambda_{t+1}) = \frac{\lambda_t}{P_{1t}} \\
&(d)\quad \beta E(\frac{e_{t+1}}{P_{1,t+1}}\lambda_{t+1}) = \frac{\lambda_t e_t}{P_{1t}}
\end{aligned} \tag{4.39}
$$

From equations (4.39a) and (4.39b), the marginal rate of substitution between domestic and foreign consumption goods is the price ratio,

$$
\frac{\frac{\partial U}{\partial C^1_{1t}}}{\frac{\partial U}{\partial C^2_{1t}}} = \frac{P_{1t}}{e_t P_{2t}}
$$

It is the equilibrium condition for consumption allocations on domestic and foreign goods.

Equation (4.39c) is the first-order difference equation in λ_t. The solution of λ_t can be solved as a function of the discounted factor, β, the expected and current prices, and probability distribution of output.

There is one interesting finding from equations (4.39c) and (4.39d) that the exchange rate is constant over time in the case that the cash-in-advance constraint is nonbinding in a certainty world. This result comes out to be the same as that obtained from a two-country version of an overlapping generations model with laissez-faire currency portfolio[3].

4.1.2.2 The solution of the optimization problem We shall solve the optimization problem to get at the explicit solutions of endogenous variables by specifying a functional form of utility function. The natures of the demand functions for domestic and foreign goods, and domestic and foreign currencies are attempted to be examined with this utility function form.

Assume that the current utility function is assumed to

$$U_t = \frac{(C^1_{1t} C^1_{2t})^{\frac{\theta}{2}}}{\theta}, \qquad \theta > 0 \tag{4.40}$$

where $1 - \theta$ is the measure of relative risk aversion. By equations (4.32a)–(4.32d), the necessary conditions become

$$(a) \quad \frac{1}{2}(C^1_{1t})^{\frac{\theta}{2}-1}(C^1_{2t})^{\frac{\theta}{2}} = \lambda_t + \psi_t$$

$$(b) \quad \frac{1}{2}(C^1_{1t})^{\frac{\theta}{2}}(C^1_{2t})^{\frac{\theta}{2}-1} = \lambda_t \frac{e_t P_{2t}}{P_{1t}} + \psi_t$$

[3]Kareken and Wallace (1981) studied how the exchange rate determined in a two-country version of an overlapping generations model with free trade in goods, loans and currencies, and found the indeterminancy of the exchange rate, that is, the exchange rate is constant over time.

$$
\begin{aligned}
&(c) \quad \beta E\left[\frac{1}{P_{1,t+1}}\frac{1}{2}(C^1_{1,t+1})^{\frac{\theta}{2}-1}(C^1_{2,t+1})^{\frac{\theta}{2}}\right] = \frac{\lambda_t}{P_{1t}} \\
&(d) \quad \beta E\left[\frac{1}{P_{2,t+1}}\frac{1}{2}(C^1_{1,t+1})^{\frac{\theta}{2}}(C^1_{2,t+1})^{\frac{\theta}{2}-1}\right] = \frac{\lambda_t e_t}{P_{1t}} \\
&(f) \quad \frac{M^1_{1,t-1}}{P_{1t}} - C^1_{1t} \geq 0, \qquad \psi_t \geq 0, \qquad (\frac{M^1_{1,t-1}}{P_{1t}} - C^1_{1t})\psi_t = 0 \\
&(g) \quad \frac{M^1_{2,t-1}}{P_{2t}} - C^1_{2t} \geq 0, \qquad \nu_t \geq 0, \qquad (\frac{M^1_{2,t-1}}{P_{2t}} - C^1_{2t})\nu_t = 0 \qquad (4.41)
\end{aligned}
$$

If all the cash-in-advance constraints are binding for the representative consumers in both countries, then the current consumptions in both countries are given by

$$
\begin{aligned}
&(a) \quad C^1_{1t} = \frac{M^1_{1,t-1}}{P_{1t}}, \qquad C^1_{2t} = \frac{M^1_{2,t-1}}{P_{1t}} \\
&(b) \quad C^2_{1t} = \frac{M^2_{1,t-1}}{P_{2t}}, \qquad C^2_{2t} = \frac{M^2_{2,t-1}}{P_{2t}} \qquad (4.42)
\end{aligned}
$$

The solutions of C^1_{1t}, C^1_{2t}, C^2_{1t}, and C^2_{2t} must satisfy the goods market clearings

$$
\begin{aligned}
&(a) \quad C^1_{1t} + C^2_{1t} = y_{1t} \\
&(b) \quad C^1_{2t} + C^2_{2t} = y_{2t} \qquad (4.43)
\end{aligned}
$$

Equation (4.43a) is the market clearings condition of goods 1 that the total demand for goods one is equal to the output of domestic goods 1. Equation (4.43b) is the market clearings condition of goods 2 that total demand for good 2 is equal to the output of foreign goods 2.

It can be apprehended that in this general equilibrium framework of two-country world the domestic and foreign demand for domestic and foreign currency must satisfy the following conditions.

1. money market clearings conditions

$$(a)\quad m^1_{1t} + m^2_{1t} = \overline{m}_{1t}$$

$$(b)\quad m^1_{2t} + m^2_{2t} = \overline{m}_{2t} \tag{4.44}$$

2. budget constraints

$$(a)\qquad m^1_{1t} + \frac{e_t P_{2t}}{P_{1t}} m^1_{2t} = y_{1t} + (\overline{m}_{1t} - \overline{m}_{1,t-1}\frac{P_{1,t-1}}{P_{1t}})$$

$$(b)\quad m^2_{1t} + \frac{e_t P_{2t}}{P_{1t}} m^2_{2t} = \frac{P_{2t}}{P_{1t}} y_{2t} + \frac{e_t}{P_{1t}}(\overline{m}_{2t}P_{2t} - \overline{m}_{2,t-1}P_{2,t-1}) \tag{4.45}$$

3. necessary conditions for utility maximization for domestic consumer from equations (4.41a)-(4.41d), and the corresponding necessary conditions for the foreign consumer

$$(a)\quad \beta E[(\frac{P_{1t}}{P_{1,t+1}}\frac{P_{2t}}{P_{2,t+1}} m^1_{1t} m^1_{2t})^{\frac{\theta}{2}}] = (\frac{P_{1,t-1}}{P_{1t}} m^1_{1,t-1} m^1_{2,t-1})^{\frac{\theta}{2}} - \psi_t$$

$$(b)\quad \beta E[(\frac{P_{1t}}{P_{1,t+1}}\frac{P_{2t}}{P_{2,t+1}} m^2_{1t} m^2_{2t})^{\frac{\theta}{2}}] = (\frac{P_{1,t-1}}{P_{1t}} m^2_{2,t-1} m^1_{2,t-1})^{\frac{\theta}{2}} - \psi_t \tag{4.46}$$

There is another interesting finding from equations (4.41c) and (4.41d) that exchange rate $e_t = 1$ with binding cash-in-advance constraints. It implies that the domestic and foreign currencies are equivalent media of exchange.

If all the cash-in-advance constraints are always nonbinding, the $\psi_t = \nu_t = 0$ for all time t. The equilibrium domestic and foreign demand for goods and currencies $(C^1_{1t}, C^1_{2t}, C^2_{1t}, C^2_{2t})$ and $(m^1_{1t}, m^1_{2t}, m^2_{1t}, m^2_{2t})$ must simultaneously satisfy the following conditions.

1. marginal rate of substitution between domestic and foreign goods consumption equates across domestic and foreign consumers

$$\frac{C^1_{2t}}{C^1_{1t}} = \frac{C^2_{2t}}{C^2_{1t}} = \frac{P_{1t}}{e_t P_{2t}} \tag{4.47}$$

2. goods market clearing conditions

$$\begin{aligned}(a) \quad & C^1_{1t} + C^2_{1t} = y_{1t} \\ (b) \quad & C^1_{2t} + C^2_{2t} = y_{2t}\end{aligned} \tag{4.48}$$

3. money market clearing conditions

$$\begin{aligned}(a) \quad & m^1_{1t} + m^2_{1t} = \overline{m}_{1t} \\ (b) \quad & m^1_{2t} + m^2_{2t} = \overline{m}_{2t}\end{aligned} \tag{4.49}$$

4. necessary conditions for the demand for domestic and foreign currency from equation (4.39c)

$$\begin{aligned}(a) \quad & \beta E[\frac{1}{P_{1,t+1}}(C^1_{1,t+1})^{\frac{\theta}{2}-1}(C^1_{2,t+1})^{\frac{\theta}{2}}] = (C^1_{1t})^{\frac{\theta}{2}-1}(C^1_{2t})^{\frac{\theta}{2}} \\ (b) \quad & \beta E[\frac{1}{P_{1,t+1}}(C^2_{1,t+1})^{\frac{\theta}{2}-1}(C^2_{2,t+1})^{\frac{\theta}{2}}] = (C^2_{1t})^{\frac{\theta}{2}-1}(C^2_{2t})^{\frac{\theta}{2}}\end{aligned} \tag{4.50}$$

5. budget constraints

$$\begin{aligned}(a) \quad C^1_{1t} \quad & + \frac{e_t P_{2t}}{P_{1t}} C^1_{2t} + m^1_{1t} + e_t m^1_{2t} \frac{P_{2t}}{P_{1t}} = y_{1t} + \\ & (\overline{m}_{1t} - \overline{m}_{1,t-1} \frac{P_{1,t-1}}{P_{1t}}) + m^1_{1,t-1} \frac{P_{1,t-1}}{P_{1t}} \\ & + e_t m^1_{2,t-1} \frac{P_{2,t-1}}{P_{1t}} \\ (b) \quad C^2_{1t} \quad & + \frac{e_t P_{2t}}{P_{1t}} C^2_{2t} + m^2_{2t} \frac{P_{2t}}{P_{1t}} = \frac{P_{2t}}{P_{1t}} y_{2t} + \frac{e_t}{P_{1t}} (\overline{m}_{2t} P_{2t} \\ & - \overline{m}_{2,t-1} P_{2,t-1}) + m^2_{1,t-1} \frac{P_{1,t-1}}{P_{1t}} \\ & + e_t m^2_{2,t-1} \frac{P_{2,t-1}}{P_{1t}}\end{aligned} \tag{4.51}$$

From the above analysis, the consumption and money demand are simultaneously determined. Consumption demand for domestic and foreign goods are functions of current prices of domestic and foreign goods, the probability distribution of domestic and foreign output, the growth rate of money supply in both countries, and the expected future prices.

4.2 The Overlapping Generations Model

In this section we shall proceed to investigate the consumption and money demand based on the insights from the overlapping generations model.

First, consider a simple economy in which money is the only way of carrying wealth from period to period. The economy produces only one nonstorable consumption goods. The bond asset will be included in our model in the subsequent section.

Each consumer lives two periods. Consumers born in period t are called young at time t and old at time $t+1$, and dead at time $t+2$ and beyond. Each country has constant population over time. At each time there are born the same number of young people. There are two types of people each time. Each member receives income endowment each period. Each consumer has identical preference.

Each consumer has the intertemporal utility function:

$$U_t = \frac{[C_t(t)C_t(t+1)]^{\frac{\theta}{2}}}{\theta}, \qquad 0 < \theta < 1 \tag{4.52}$$

where

U_t = utility of an individual born at period t

$C_t(t)$ = consumption in t of an individual born at period t

$C_t(t+1)$= consumption in $t+1$ of an individual born at period t

$1-\theta$ = the measure of relative risk aversion

We use the version of the consumption-loan model[4] to examine the consumption and savings decisions of the consumer and to derive the demand for consumption and money (portfolio allocations). In the overlapping generations model, the money is essentially a store of wealth. The currency is actually a loan to the government that issues that money. Assume that government issues new money via monetary transfer payment to the public. A lump-sum monetary transfer payment is only effected to the old generation, rather than to the young generation.

4.2.1 A simple economy with money

We consider a simple economy in which money is the only way for carrying wealth. The young generation allocates his endowment between current consumption and money holdings. The budget constraint of the young person at time t is

$$C_t(t) + \frac{M_t}{P_t} = y_t(t) \tag{4.53}$$

where

M_t = nominal money holdings of the individual from t to $t+1$

[4]Samuelson (1958) developed a consumption-loan version of overlapping generations model. Recently, OLG model has been used in many fields of economics. Many studies have addressed the question of welfare effect of money supply in the overlapping generation model (Wallace (1980), Brock and Scheinkman (1980)). Kareken and Wallace (1981 and 1978) analyze the determination of exchange rate based on the OLG model.

P_t = the price of the goods at t

$y_t(t)$ = the endowment in t of the agent born at t

In the second period of life, the consumer spends all his endowment of that period and all the other wealth. This is no banquet or bestow left over to the next generation. The consumption in $t+1$ of an individual born at t, $C_t(t+1)$, is

$$C_t(t+1) = y_t(t+1) + \frac{M_t}{P_{t+1}} + \tau_{t+1} \tag{4.54}$$

where $y_t(t+1)$ = the endowment in $t+1$ of the agent born at t

The maximization problem of the consumer born at t is to choose $C_t(t)$, $C_t(t+1)$ and M_t, subject to the constraints of equations (4.53) and (4.54), so as to maximize his lifetime utility. The Lagrangian of the optimization problem is

$$\mathcal{L} = \frac{[C_t(t)(y_t(t+1) + \frac{M_t}{P_{t+1}} + \tau_{t+1})]^{\frac{\theta}{2}}}{\theta} + \lambda_1(y_t(t) - C_t(t) - \frac{M_t}{P_t}) \tag{4.55}$$

The necessary conditions are:

$$(a) \quad \frac{\partial \mathcal{L}}{\partial C_t(t)} [C_t(t)]^{\frac{\theta}{2}-1}(y_t(t+1) + \frac{M_t}{P_{t+1}} + \tau_{t+1})^{\frac{\theta}{2}} - \lambda_1 = 0$$

$$(b) \quad \frac{\partial \mathcal{L}}{\partial M_t} = [C_t(t)]^{\frac{\theta}{2}}(y_t(t+1) + \frac{M_t}{P_{t+1}} + \tau_{t+1})^{\frac{\theta}{2}-1}\frac{1}{P_{t+1}} - \frac{\lambda_1}{P_t} = 0 \tag{4.56}$$

From the necessary conditions of equation (4.56) and budget constraint, the current and future consumption, and money demand can be easily solved as

$$(a) \quad C_t(t) = \frac{1}{2}[y_t(t) + \frac{P_{t+1}}{P_t}(y_t(t+1) + \tau_{t+1})]$$

$$(b) \quad C_t(t+1) = \frac{1}{2}[\frac{P_t}{P_{t+1}} y_t(t) + y_t(t+1) + \tau_{t+1}]$$

$$(c) \quad \frac{M_t}{P_t} = \frac{1}{2}[y_t(t) - \frac{P_{t+1}}{P_t}(y_t(t+1) + \tau_{t+1})] \tag{4.57}$$

Equation (4.57a) expresses that the current consumption depends on the present value of his total wealth over his lifetime periods. From equations (4.57a) and (4.57b), the optimal solutions of current and future consumptions satisfying the integrated constraints that the present value of total consumption during his entire life is exactly equal to the present value of his total wealth over his life periods,

$$C_t(t) + \frac{P_{t+1}}{P_t} C_t(t+1) = y_t(t) + \frac{P_{t+1}}{P_t} y_t(t+1) \tag{4.58}$$

The consumption solutions, which are consistent with the utility maximization behavior, reflect that the individual will distribute consumption over his lifetime so that he has a flat or even flow of consumption. The nominal consumption is the same in every period of his life. The consumption is not geared to current income, but rather to lifetime income.

The current consumption is half of his lifetime income and the remaining of the current income is saved in the form of money. The demand for money is given by equation (4.57c). Given other things unchanged, the increase in future price will lower money demand. The intuitive explanation is straightforward that the increases in the goods price result in the total wealth increase and lead to more current goods consumption, and therefore the money demand will decrease. Likewise, if the individual receives more monetary transfer payment in his old life period, he will spend more consumption and hold less money during his young period.

Assume that government monetary transfer payment is financed by creating new money

$$\tau_t = \left(\frac{\overline{M}_t}{P_t} - \frac{\overline{M}_{t-1}}{P_t}\right) \tag{4.59}$$

where τ_t is the amount of monetary transfer payment of the old generation in period

t, measured in real term. $\overline{M}_t$ is the total money supply in period t divided by the number of people of each generation(N). In brief, each consumer only receives the monetary transfer payment during his second period of life.

From equation (4.57), the consumption of the individual old person at time t is

$$C_{t-1}(t) = \frac{1}{2}[\frac{P_{t-1}}{P_t}y_{t-1}(t-1) + y_{t-1}(t) + (\frac{\overline{M}_t}{P_t} - \frac{\overline{M}_{t-1}}{P_t})] \tag{4.60}$$

Thus, the aggregate consumption at time t is the total sum of the consumption of young and old generations, C_t, in real term

$$\begin{aligned} C_t &= \frac{N}{2}\{[y_t(t) + y_{t-1}(t)] + \frac{P_{t+1}}{P_t}y_t(t+1) + \frac{P_{t-1}}{P_t}y_{t-1}(t-1) \\ &\quad +(\frac{\overline{M}_{t+1}}{P_t} - \frac{\overline{M}_{t-1}}{P_t})]\} \end{aligned} \tag{4.61}$$

The term of $N[y_t(t) + y_{t-1}(t)]$ in equation (4.61) is the current total income in period t. $Ny_t(t+1)$ is the total income in period $t+1$ of young generation born in period t. $Ny_{t-1}(t-1)$ is the past income in period $t-1$ of the old generation born in period $t-1$. Equation (4.61) expresses that the aggregate consumption depends on the current total income, some of past income, some of future income and the lagged money supply and next period income, and current price, lagged price and expected future price.

The total money demand at time t is the sum of the money holdings of the young generation born at t

$$N\frac{M_t}{P_t} = \frac{1}{2}[Ny_t(t) - \frac{P_{t+1}}{P_t}Ny_t(t+1) - \frac{1}{P_t}(\overline{M}_{t+1} - \overline{M}_t)] \tag{4.62}$$

The demand for money depends on (1) the total current and future income of young generations, (2) current and next period money supply, (3) current and future price.

4.2.2 An economy with bond

We extend the overlapping generations model in the preceding section to analyze the consumption and portfolio decisions of the individual behavior by including government bond in monetary assets.

The budget constraint of the young person at time t is

$$C_t(t) + \frac{M_t}{P_t} + \frac{B_t}{P_t(1+r_t)} = y_t(t) \tag{4.63}$$

where

B_t = the quantity of the nominal bond that the individual holds from t to $t+1$

r_t = the nominal interest rate in period t

The consumption in $t+1$ of an individual born at t, $C_t(t+1)$ is

$$C_t(t+1) = y_t(t+1) + \frac{M_t}{P_{t+1}} + \frac{B_t}{P_{t+1}} + \tau_{t+1} \tag{4.64}$$

The maximization problem of the consumer born at t is to choose $C_t(t)$, $C_t(t+1)$, M_t and B_t, subject to the constraints of equations (4.60) and (4.61), and given prices, P_t and P_{t+1}, and endowment, $y_t(t)$ and $y_t(t+1)$, so as to maximize his lifetime utility. The Lagrangian of the optimization problem is

$$\begin{aligned} \mathcal{L} \;=\; & \frac{[C_t(t)C_t(t+1)]^{\frac{\theta}{2}}}{\theta} + \lambda_1[y_t(t) - C_t(t) - \frac{M_t}{P_t} - \frac{B_t}{P_t(1+r_t)}] \\ & +\lambda_2[y_t(t+1) + \frac{M_t}{P_{t+1}} + \frac{B_t}{P_{t+1}} + \tau_{t+1} - C_t(t+1)] \end{aligned} \tag{4.65}$$

The necessary conditions are:

$$(a) \quad \frac{\partial \mathcal{L}}{\partial C_t(t)} = \frac{[C_t(t)]^{\frac{\theta}{2}-1}[C_t(t+1)]^{\frac{\theta}{2}}}{2} - \lambda_1 = 0$$

$$(b)\quad \frac{\partial \mathcal{L}}{\partial C_t(t+1)} = \frac{[C_t(t)]^{\frac{\theta}{2}}[C_t(t+1)]^{\frac{\theta}{2}-1}}{2} - \lambda_2 = 0$$

$$(c)\quad \frac{\partial \mathcal{L}}{\partial M_t} = \frac{-\lambda_1}{P_t} + \frac{\lambda_2}{P_{t+1}} \leq 0, \qquad M_t \frac{\partial \mathcal{L}}{\partial M_t} = 0$$

$$(d)\quad \frac{\partial \mathcal{L}}{\partial B_t} = \frac{-\lambda_1}{(1+r_t)P_t} + \frac{\lambda_2}{P_{t+1}} \leq 0, \qquad B_t \frac{\partial \mathcal{L}}{\partial B_t} = 0$$

$$(e)\quad \frac{\partial \mathcal{L}}{\partial \lambda_1} = y_t(t) - C_t(t) - \frac{M_t}{P_t} - \frac{B_t}{P_t(1+r_t)} = 0$$

$$(f)\quad \frac{\partial \mathcal{L}}{\partial \lambda_2} = y_t(t+1) + \tau_t(t+1) + \frac{M_t}{P_{t+1}} + \frac{B_t}{P_{t+1}} - C_t(t+1) = 0 \quad (4.66)$$

From the necessary conditions of equations (4.66a)-(4.66c), we have that

$$\frac{C_t(t+1)}{C_t(t)} = \frac{P_t}{P_{t+1}} \tag{4.67}$$

Equation (4.67) expresses that the marginal rate of substitution is equal to the intertemporal price ratio. The consumer optimally allocates current and future consumptions reaching up to the point where the nominal value of consumption is the same in the two periods of his life.

From equations (4.66a) and (4.66b), the arbitrage condition of money and bond requires that the nominal interest rate must be zero. There is no existence of positive nominal interest rate if demand for money and bond coexists. Intuitively, if the nominal interest rate is positive, the consumer can claim one-dollar realized value from one-period bond tomorrow by only paying the cost less than one dollar today. In other words, the discounted present value of one-dollar bond due on tomorrow is less than one dollar if the nominal interest rate is positive, *i.e.*, $\frac{1}{(1+r_t)} < 1$. The consumer won't hold money if the nominal interest rate is positive. It is clear that the currency and bond are two equivalent interest-free monetary assets in equilibrium in this overlapping generations model if no default risk is involved.

The current and future optimal consumption, and savings decisions are obtained from the necessary conditions and the budget constraints that

$$
\begin{aligned}
&(a) \quad C_t(t) = \tfrac{1}{2}[y_t(t) + \frac{P_{t+1}}{P_t}(y_t(t+1) + \tau_{t+1})] \\
&(b) \quad C_t(t+1) = \tfrac{1}{2}[\frac{P_t}{P_{t+1}} y_t(t) + y_t(t+1) + \tau_{t+1}] \\
&(c) \quad \frac{M_t}{P_t} + \frac{B_t}{P_t} = \tfrac{1}{2}[y_t(t) - \frac{P_{t+1}}{P_t}(y_t(t+1) + \tau_{t+1})] \qquad (4.68)
\end{aligned}
$$

Equation (4.68a) indicates that the current consumption depends on the discounted present value of all his income over his lifetime periods. The value of $\frac{P_{t+1}}{P_t}$ is the discounted factor as capitalizing the future income. From equations (4.68a) and (4.68b), the optimal solutions of current and future consumptions satisfy the integrated budget constraints that the present value of total consumption in life is exactly equal to the present value of the total wealth in his entire life,

$$
C_t(t) + \frac{P_{t+1}}{P_t} C_t(t+1) = y_t(t) + \frac{P_{t+1}}{P_t}[y_t(t+1) + \tau_{t+1}]
$$

The aggregate consumption demand at t is the sum of the consumption of young and old persons, C_t

$$
C_t = \frac{N}{2}[y_t(t) + \frac{P_{t+1}}{P_t} y_t(t+1) + \frac{P_{t-1}}{P_t} y_{t-1}(t-1) + \frac{\overline{M}_{t+1}}{P_t} - \frac{\overline{M}_{t-1}}{P_t}] \qquad (4.69)
$$

Equation (4.69) expresses that aggregate consumption is a function of current income, young generation's future income and old person's previous income, current price, lagged and future price, and lagged and future money supply. The aggregate consumption demand can be approximated as a function of current income and price, lagged income and price, and future income and price.

At each time the total demand for money is simply the sum of money demand of all the young persons. Thus, the total demand for money is calculated as

$$N\frac{M_t}{P_t} = \frac{(1-s)}{2}\left[Ny_t(t) - \frac{P_{t+1}}{P_t}Ny_t(t+1) - N\left(\frac{\overline{M}_{t+1}}{P_t} - \frac{\overline{M}_t}{P_t}\right)\right] \qquad (4.70)$$

The total demand for money is a function of total current and future income of the young people, current and future money supply, and current and future price.

The demand for consumption and real balance can be linearized as the functions of the forms

$$\begin{aligned}
(a)\ C_t &= a_1 y_{t-1} + a_2 y_t + a_3 E_t y_{t+1} + a_4 P_{t-1} + a_5(E_t P_{t+1} - P_t) \\
&\quad + a_6 M_{t+1} + a_7 M_{t-1} \\
(b)\ \frac{M_t}{P_t} &= b_1 y_t - b_2 E_t y_{t+1} - b_3(E_t P_{t+1} - P_t) \\
&\quad - b_4(E_t M_{t+1} - E_t P_{t+1}) \qquad (4.71)
\end{aligned}$$

where a's and b's are positive. The sign of the term $(E_t P_{t-1} - P_t)$ is positive in consumption but negative in money demand functions. The reason is straightforward that the higher expected price means the higher present value of lifetime income. It follows that the demand for consumption and money will increase under the assumption of normal goods. There is a plausible result from the overlapping generations model that the expected lifetime income is a determinant variable appearing in the consumption function.

5 EXTENSIONS

We will develop two revised stochastic open-macroeconomic models based on insights from the consumer optimization problems of the cash-in-advance and overlapping generations models. The demand functions for money and consumption in the T-B model will be replaced by our revised functions,which were derived in the previous chapter. The main concern in this chapter continues to be the issue of the optimal money supply rule for these two revised models. The purpose of this chapter is to examine whether the structure of the economy does really matter in the choice of optimal monetary policy.

Unfortunately, the optimal feedback money-supply rule does not exist in either of the two revised macroeconomic models. In both models, rational expectations are assumed, the public and the monetary authority having the same information set. Moreover, no rational expectation solution of price can be found when Murata's control method is applied to these two revised models in deriving the feedback rule. Doing so yields the Sargent-Wallace result, which provides important support for Friedman's contention that the monetary authority should abandon attempts to pursue an activist stabilization policy.

The failure of Murata's control method when applied to the revised models inspires the search for an alternative optimal money-supply rule for an economy where the monetary authority has information superior to that of the public. The public forms its expectation of the event of time t at time $t-1$. But the monetary authority has full knowledge of all current random disturbances when he sets the money-supply rule for time t.

In this chapter, we will derive an alternative optimal money-supply rule to the usual feedback rule, under the assumption that the monetary authority has information superior to that of the public. It provides the support for the effectiveness of stabilization policy.

5.1 A Revised Model – Cash-in-advance Model

In the cash-in-advance model, the demand for money is determined primarily by the purpose of the transaction. It can be inferred from an analysis of the cash-in-advance model that the transaction demand for money is only a function of current income and in fact has nothing to do with the interest rate. In addition, the consumption demand function is a function of current income and expected inflation rate. The demand functions for money and consumption are expressed in the linear forms

$$(a) \qquad m_t - p_t = \beta_1 y_t, \quad \beta_1 > 0 \quad and$$

$$(b) \quad c_t = c_1 y_t + c_2(E_t p_{t+1} - p_t) + w_t \qquad c_1 > 0\ , \qquad c_2 > 0 \qquad (5.1)$$

Equation (5.1a) and (5.1b) replace the money-demand equation (2.2) and consumption-demand equation (2.7) in the T-B model. The original aggregate supply equation (2.1) is slightly changed by including lagged income. One revised model is developed and reduced to the following four equations:

$$(a) \qquad y_t = s_1 y_{t-1} + s_2(p_t - E_{t-1}p_t) + u_t$$

$$(b) \qquad m_t - p_t = \beta_1 y_t$$

$$(c) \qquad p_t = p_t^* + e_t \qquad and$$

$$(d) \quad \eta[(E_t e_{t+1} - E_{t-1}e_t) - (e_t - e_{t-1} + (r_t^* - r_{t-1}^*) - (r_t - r_{t-1})] = (1 - c_1)y_t - c_2(E_t p_{t+1} - p_t) - w_t \tag{5.2}$$

The system is recursive since equation (5.2a) and (5.2b) simultaneously and uniquely determine the time paths of income and price. Then, the exchange rate is determined by equation (5.2c). Once the solutions of y_t, p_t, and e_t are obtained, the interest rate is subsequently determined by equation (5.2d). The recursive nature of the system calls for the subsystem, rather than the whole system, to be controlled. More specifically, the policy instrument of money supply would affect price and output by way of the money market, along with the domestic aggregate supply.

5.1.1 The superior-information money rule

Assuming that the monetary authority has information superior to that of the public, the groups do not share the same information at any time when forming expectations rationally. The monetary authority has full information about current random shocks arising from the domestic supply (u_t), demand sides (w_t), and foreign prices (p_t^*). The current money supply rule is stated in the general form

$$m_t = \overline{m} + a_1 u_t + a_2 p_t^* + a_3 w_t \tag{5.3}$$

where $\overline{m}$ is the time trend of money supply, which is a component of money anticipated by the public. The logic behind equation (5.3) is that the government attempts to offset the impact of random shocks on the economy by adopting a permissible money-supply rule, as long as it has better knowledge of information than the public does. In the case of superior-information rational expectations, the economy may be completely insulated from certain random shocks. The goal of the government is to minimize the variance of the variable concerned (income or price) by choosing the coefficients of a_1, a_2, and a_3, which are associated with all of the random shocks in the system.

According to the money-supply rule described in (5.3), the rational expectation value of m_t formed at time $t-1$ by the public is

$$E_{t-1}m_t = \overline{m} \tag{5.4}$$

The rational expectation value of money supply is exactly the time trend of money supply. Since the public has no information about time t random shocks when it forms its expectations at time $t-1$, clearly $E_{t-1}u_t = E_{t-1}p_t^* = E_{t-1}w_t = 0$. It follows from (5.2a) and (5.2b) that

$$\begin{aligned} E_{t-1}y_t &= s_1 y_{t-1} \qquad \textit{and} \\ E_{t-1}m_t &= E_{t-1}p_t + s_1\beta_1 y_{t-1} \end{aligned} \tag{5.5}$$

From (5.4) and (5.5), the expectation value of p_t formed at $t-1$ by the public is

$$E_{t-1}p_t = \overline{m} - s_1\beta_1 y_{t-1} \tag{5.6}$$

The pseudo-reduced form of y_t from (5.2a), (5.2b), (5.3), and (5.6) is written as

$$y_t = s_1 y_{t-1} + \frac{(1+s_2a_1)}{(1+s_2\beta_1)}u_t + \frac{s_2a_2}{1+s_2\beta_1}p_t^* + \frac{s_2a_3}{1+s_2\beta_1}w_t \tag{5.7}$$

Suppose that the goal of the government is to minimize the variance of income, or the sum of the squares of income deviations, from its targeted value. Therefore, the values of a_1, a_2, and a_3 must be chosen as

$$a_1 = -\frac{1}{s_2}, \quad a_2 = 0, \quad a_3 = 0, \tag{5.8}$$

such that the asymptotic variance of y_t attains the minimum value. The money-supply rule incorporating superior information and rational expectations is expressed as

$$m_t = \overline{m} - \frac{1}{s_2}u_t \tag{5.9}$$

where $\overline{m}$ is the time trend of money supply For convenience, set $\overline{m}$ equal to zero.

The reduced forms of y_t, p_t, and e_t under this superior-information rule (5.9) are easily obtained:

$$\begin{aligned} &(a) \quad y_t = s_1 y_{t-1} \\ &(b) \quad p_t = \overline{m} - \frac{1}{s_2}u_t - s_1\beta y_{t-1} \qquad and \\ &(c) \quad e_t = \overline{m} - \frac{1}{s_2}u_t - s_1\beta_1 y_{t-1} - p_t^* \end{aligned} \tag{5.10}$$

The time path of income under this money-supply rule (5.9) is a first-order autoregressive process involving no random disturbances. Under this rule, no serial correlation in income has been eliminated. Since the money-supply rule here has been designed in an attempt to eliminate random shocks arising from aggregate supply, the income process can be fully insulated from random supply shocks (u_t).

The condition necessary for this first-order income process to be convergent is the value s_1 being less than one ($0 < s_1 < 1$). The time paths of price and exchange rate of equations (5.10b) and (5.10c) are the stochastic processes of a one-lagged income with some random disturbances (u_t, p_t^*).

Substituting equations (5.10) and (5.6) into (5.2d), we have the reduced form of interest rate:

$$\begin{aligned} r_t &= r_{t-1} + \{s_1\beta_1[1 + (1 - s_1)(1 + \frac{c_2}{\eta})] - s_1(\frac{1 - c_1}{\eta})\}y_{t-1} - s_1\beta_1 y_{t-2} \\ &+(1 + \frac{c_2}{\eta})\frac{1}{s_2}u_t - \frac{1}{s_2}u_{t-1} + (p_t^* - p_{t-1}^*) + (r_t^* - r_{t-1}^*) + \frac{w_t}{\eta} \qquad (5.11) \end{aligned}$$

The first-order difference process for interest rate is an autoregressive and moving average process of income, price, and random domestic supply disturbances with all other disturbance terms.

According to equation (5.10), the asymptotic variance of y_t depends on the response of the aggregate output to the lagged income (s_1) and the initial value of income. The asymptotic variance of p_t and e_t are easily obtained:

$$\begin{aligned} \sigma_p^2 &= (\frac{1}{s_2})^2\sigma_u^2 + (s_1\beta_1)^2\sigma_y^2 \\ \sigma_e^2 &= (\frac{1}{s_2})^2\sigma_u^2 + (s_1\beta_1)^2\sigma_y^2 + \sigma_{p*}^2 \qquad (5.12) \end{aligned}$$

where σ_p^2, σ_e^2 and σ_y^2 are the asymptotic variances of p_t, e_t and y_t. The asymtotic variance of first-order difference ($r_t - r_{t-1}$) is

$$\begin{aligned} \sigma_{r_{-1}}^2 &= \{\{s_1\beta_1[1 + (1 - s_1)(1 + \frac{c_2}{\eta})] - s_1(\frac{1 - c_1}{\eta})\}^2 + \beta_1^2\}\sigma_y^2 \\ &+\frac{1}{s_2}[(1 + \frac{c_2}{\eta})^2 + 1]\sigma_u^2 + 2\sigma_{p*}^2 + 2\sigma_{r*}^2 + \frac{\sigma_w^2}{\eta} \qquad (5.13) \end{aligned}$$

where $\sigma_{r_{-1}}^2$ is the variance of the first-order difference $r_t - r_{t-1}$.

In this system, variation of interest rate arises from the fluctuation of income and all domestic and foreign disturbances. The degree of capital mobility affects only the fluctuations of the interest rate. Capital mobility appears to have the function of lowering the impact of domestic supply and demand shocks on the variation of interest rate.

Next, suppose that the authority desires to set m_t so as to minimize the variance of price or the forecast error of price, $E_{t-1}(p_t - \overline{p})^2$, or $E_{t-1}(p_t - E_{t-1}p_t)^2$, where $\overline{p}$ is the targeted value of price. Following the above procedure, the optimal money-supply rule can be easily derived. Equations (5.2a), (5.2b) and (5.3) can be reduced to

$$p_t = \overline{m} - s_1\beta_1 y_{t-1} + \frac{a_1 - \beta_1}{1 + s_2\beta_1} u_t + \frac{a_2}{1 + s_2\beta_1} p_t^* + \frac{a_3}{1 + s_2\beta_1} w_t \,. \tag{5.14}$$

The optimal money-supply rule is obtained by choosing $a_1 = \beta_1$, $a_2 = 0$, and $a_3 = 0$ to minimize price variance. Therefore, the optimal money-supply rule is expressed as

$$m_t = \overline{m} + \beta_1 u_t \tag{5.15}$$

Under this optimal money rule (5.15), we have the reduced form of y_t, p_t, and e_t by substituting equation (5.15) into (5.7) and (5.14):

$$\begin{aligned} y_t &= s_1 y_{t-1} + u_t \\ p_t &= \overline{m} - s_1\beta_1 y_{t-1} \qquad \textit{and} \\ e_t &= \overline{m} - s_1\beta_1 y_{t-1} - p_t^* \end{aligned} \tag{5.16}$$

The stochastic process for income is a first-order process and involves a white noise u_t. It differs from cases in which minimization of variance of income is a criterion. Random supply shock does not enter in the final forms of p_t and e_t, but does enter y_t. If the money-supply rule is set to minimize price variation by eliminating the impact of domestic supply shock, then the time paths of p_t and e_t can be totally insulated from domestic supply shock (u_t). The autoregressive income process is now affected by random supply shock, however.

Substituting equations (5.16) and (5.6) into (5.2d), we determine the reduced form of interest rate:

$$\begin{aligned} r_t &= r_{t-1} + \{s_1\beta_1[1+(\frac{c_2}{\eta})(1-s_1)] - \frac{1-c_1}{\eta}\}y_{t-1} - s_1\beta_1 y_{t-2} \\ &\quad -[(1+\frac{c_2}{\eta})s_1\beta_1 + \frac{1-c_1}{\eta}]u_t + (p_t^* - p_{t-1}^*) + (r_t^* - r_{t-1}^*) + \frac{w_t}{\eta} \quad (5.17) \end{aligned}$$

The stochastic process for interest rate (5.17) is similar to (5.11), with the exception of the random supply shock term (u_t). Lagged random supply shock (u_t) has no impact on the current interest rate, unlike results with the objective of income-variance minimization. As the degree of capital mobility increases, the fluctuation of interest rate originating from current domestic supply disturbance will increase, but that originating form current domestic demand disturbance will decrease.

The asymptotic variances of y_t, p_t, and $(r_t - r_{t-1})$ are calculated from equations (5.16) and (5.17) as

$$\begin{aligned} \sigma_y^2 &= \frac{1}{1-s_1^2}\sigma_u^2 \\ \sigma_p^2 &= \frac{(s_1\beta_1)^2}{1-s_1^2}\sigma_u^2 \\ \sigma_e^2 &= \frac{(s_1\beta_1)^2}{1-s_1^2}\sigma_u^2 + \sigma_{p^*}^2 \qquad and \\ \sigma_{r_{-1}}^2 &= \{[(1+\frac{c_2}{\eta})(1-s_1)s_1\beta_1 + s_1\beta_1 - \frac{1-c_1}{\eta}]^2 + (s_1\beta_1)^2\}/(1-s_1^2) \\ &\quad +[(1+\frac{c_2}{\eta})s_1\beta_1 + \frac{1-c_1}{\eta}]^2\}\sigma_u^2 + 2\sigma_{p^*}^2 + 2\sigma_{r^*}^2 + \frac{1}{\eta^2}\sigma_w^2 \qquad (5.18) \end{aligned}$$

5.1.2 Fixed money supply rule

Assume that monetary authority sets the simple money-supply rule

$$m_t = g \qquad \forall t,$$

where g is constant. The pseudo-reduced form of income is

$$y_t = \frac{1}{1+s_2\beta_1}(s_1 y_{t-1} + s_2 g - s_2 E_{t-1}p_t + u_t) \tag{5.19}$$

Substituting equation (5.19) into (5.2b), the pseudo-reduced form of price is

$$p_t = \frac{1}{1+s_2\beta_1}(g - s_1\beta_1 y_{t-1} + s_2\beta_1 E_{t-1}p_t - \beta_1 u_t) \tag{5.20}$$

The rational expectation solution of price, $E_{t-1}p_t$, can be easily solved as

$$E_{t-1}p_t = g - s_1\beta_1 y_{t-1} \tag{5.21}$$

Substituting equation (5.21) into (5.19) and (5.20), the reduced forms of income and price are

$$y_t = s_1 y_{t-1} + \frac{1}{1+s_2\beta_1}u_t \qquad \textit{and} \tag{5.22}$$

$$p_t = g - s_1\beta_1 y_{t-1} - \frac{\beta_1}{1+s_2\beta_1}u_t \tag{5.23}$$

As with the superior-information case, which includes the criterion of minimum price variance, income is an autoregressive process and contains a random supply disturbance term. It does not contain white noise, however. Obviously the coefficient of the term u_t in (5.22) is less than one, so the income process under the fixed money-supply rule appears to be less variate than under the former case if the economy undergoes the impact of supply shock. Price is an autoregressive process of y_t and contains a random disturbance term involving u_t. Unlike the case of superior-information with the objective of minimum price variance, price cannot be completely insulated from random supply shock u_t. But it appears to be similar to the superior-information case in that it maintains the objective of minimum income variance. The fluctuation of y_t originating from u_t under the fixed money-supply rule will be less than under the superior-information case with minimum income variance since $1/s_2 > \beta_1/(1+s_2\beta_1)$.

The reduced-form of interest rate, r_t, is obtained by substituting (5.2a), (5.2b) and (5.2c) into (5.2d):

$$\begin{aligned} r_t &= r_{t-1} + [(1+\frac{c_2}{\eta})s_1\beta_1(1-s_1) + s_1\beta_1 - (\frac{1-c_1}{\eta})s_1]y_{t-1} - s_1\beta_1 y_{t-2} \\ &+ \frac{\beta_1(1-s_1)(1+\frac{c_2}{\eta})}{1+s_2\beta_1}u_t - \frac{\beta_1}{1+s_2\beta_1}u_{t-1} \\ &+(p_t^* - p_{t-1}^*) + (r_t^* - r_{t-1}^*) + \frac{w_t}{\eta} \end{aligned} \tag{5.24}$$

Like the superior-information case with the objective of minimum income variance, the first-order difference process for interest rate $(r_t - r_{t-1})$ is an autoregressive process of the endogenous variable y_t and of the exogenous variables u_t, p_t^*, r_t^* and contains a random disturbance term involving the domestic demand shock (w_t).

The asymptotic variance of y_t, p_t, and e_t can be easily calculated as

$$\begin{aligned} \sigma_y^2 &= \frac{1}{(1-s_1^2)(1+s_2\beta_1)^2}\sigma_u^2 \\ \sigma_p^2 &= \frac{\beta_1^2(1+2s_1^2)}{(1-s_1^2)(1+s_2\beta_1)^2}\sigma_u^2 \quad \textit{and} \\ \sigma_e^2 &= \frac{\beta_1^2(1+2s_1^2)}{(1-s_1^2)(1+s_2\beta_1)^2}\sigma_u^2 + \sigma_{p^*}^2 \end{aligned} \tag{5.25}$$

The asymptotic variance of the first-order difference interest rate $(r_t - r_{t-1})$ is

$$\sigma^2_{r_{-1}} = \{\frac{[(1+\frac{c_2}{\eta})s_1\beta_1(1-s_1)+s_1\beta_1-(\frac{1-c_1}{\eta})s_1]^2+(s_1\beta_1)^2}{(1-s_1^2)(1+s_2\beta_1)^2}$$
$$+[\frac{\beta_1(1-s_1)(1+\frac{c_2}{\eta})}{1+s_2\beta_1}]^2$$
$$+(\frac{\beta_1}{1+s_2\beta_1})^2\}\sigma_u^2+2\sigma^2_{p*}+2\sigma^2_{r*}+\frac{\sigma_w^2}{\eta^2} \qquad (5.26)$$

The asymptotic variances of endogenous variables in (5.25) and (5.26) are useful when examining the sources of the fluctuations of endogenous variables and tracing the long-run effects of random disturbances on fluctuations of endogenous variables.

There is a common characteristic: with the above optimal money-supply rules of equations (5.9) and (5.15), as with the fixed money-supply rule, it is not possible to eliminate or lower the business cycle. Keep in mind that there exists no optimal feedback money-supply rule in this model. The relative performances of the above three alternative money-supply rules are examined and compared on the basis of the degree of insulation from random shocks. Therefore, the relative insulation of endogenous variables from random shocks under the alternative money-supply rules can be determined from the relative coefficients associated with the corresponding random disturbances. Under the alternative money-supply rules, the responses of variations of endogenous variables y_t, p_t, e_t, and r_t with respect to random shock u_t are described in Table 5.1.

In Table 5.1, (*) and (**) are defined as follows:

$$(*): \qquad \frac{[(1+\frac{c_2}{\eta})(1-s_1)s_1\beta_1+s_1\beta_1-\frac{1-c_1}{\eta}]^2}{1-s_1^2}+[(1+\frac{c_2}{\eta})s_1\beta_1+\frac{1-c_1}{\eta}]^2$$

Table 5.1: The responses of variation of endogenous variable to current shocks u_t under random shocks according to the alternative money supply rules

	the minimun income variance rule	the minimun price variance rule	the fixed rule
y_t	0	1	$\frac{1}{(1-s_1^2)(1+s_2\beta_1)^2}$
p_t	$(\frac{1}{s_2})^2$	$\frac{(s_1\beta_1)^2}{1-s_1^2}$	$\frac{\beta_1^2(1+2s_1)^2}{(1-s_1^2)(1+s_2\beta_1)^2}$
e_t	$(\frac{1}{s_2})^2$	$\frac{(s_1\beta_1)^2}{1-s_1^2}$	$\frac{\beta_1^2(1+2s_1)^2}{(1-s_1^2)(1+s_2\beta_1)^2}$
r_t	$\frac{1}{s_1^2}[(1+\frac{c_2}{\eta})^2+1]$	(*)	(**)

$$(**): \quad \frac{[(1+\frac{c_2}{\eta})(1-s_1)s_1\beta_1+s_1\beta_1-\frac{1-c_1}{\eta}s_1]^2+(s_1\beta_1)^2}{(1-s_1^2)(1+s_2\beta_1)^2}+\frac{[\beta_1(1-s_1)(1+\frac{c_2}{\eta})]^2+\beta_1^2}{(1+s_2\beta_1)^2}$$

Income under the rule with the minimum-income-variance criterion can be completely insulated from domestic supply shock. But income under both the rule with the minimum-price-variance criterion and the fixed rule cannot be insulated from the impact of domestic supply shock. The quantitative impact on the variation of income under the minimum-price-variance rule is exactly like that of one independent of any structural coefficients. Under the fixed rule, however, variation of income originating from domestic supply shock depends on the structural coefficients of the domestic supply side and the money market. The impact on income under the minimum-price-variance rule is smaller than that of the fixed money supply rule if $(1-s_1^2) < 1/(1+s_2\beta_1)^2$.

It can be inferred that if the economy exhibits a high speed of adjusting price forecast error and high income elasticity of money demand, the money supply rule with the minimum-price-variance criterion will be more powerful than a fixed money supply in lessoning the income fluctuation resulted from domestic supply shock.

A comparisons of the relative performances of the three alternative money supply rules, from the standpoint of smallest price variation, suggests that the optimal rule depends on the quantitative magnitudes of the structural coefficients. Under the rule with the minimum income variance criterion, the price variation resulting from domestic supply shock depends solely on the response of y_t with respect to the price forecast error. Under this minimum price-variance rule, the price and exchange rate varies less if the aggregate supply is less responsive to price forecast error. In contrast, the fluctuations of price and exchange rate resulting from domestic supply shock under the minimum-price-variance rule depends on the responsiveness of aggregate supply to lagged output (s_1) and the income elasticity of money demand (β_1). In this case, the price and exchange rates will vary more as the responsiveness of aggregate supply to lagged output increases.

Under the fixed money-supply rule, price and exchange rate variations originating from u_t are determined by the structural coefficients of the supply side (s_1, s_2) and the income elasticity of the money demand (β_1). As with the minimum-price-variance rule, the fluctuation of price originating from u_t under the fixed rule will be larger as the aggregate output responds more to the lagged output. If $s_2\beta_1 > (1 + s_1)/s_1$, then the price and exchange rate will demonstrate more variation originating from u_t under the minimum-price-variance rule than from under the fixed rule. It appears that the fixed money-supply rule is more powerful in attacking the impact of domestic supply shock on the variations of price and exchange rates.

Lastly, the relative insulation of the interest rate from u_t under the alternative money-supply rules is easily worked on from Table 5.1. Under the rule with the minimum-income-variance criterion the fluctuation of interest rate originating from u_t depends on the responsiveness of aggregate supply to lagged output, the elasticity of consumption with respect to the expected inflation, and the degree of capital mobility. The interest rate will vary less as the responsiveness of current aggregate supply to lagged output increases. Notice that the elasticity of aggregate supply with respect to price forecast error has nothing to do with the variations of any endogenous variable as a result of random supply shock.

Under the rule with the minimum-price-variance criterion and under the fixed rule, variation of interest rate originating from u_t will be larger as the responsiveness of aggregate supply to lagged output and income elasticity of money demand increases. All the structural coefficients of both the supply and demand sides will affect the variation of interest rate resulting from domestic supply shock. It is very complicated to compare the relative variations of interest rates under alternative money supply rules. But it is simpler to examine the time path of interest rates under the alternative money-supply rules, once the values of all the structural coefficients have been given.

5.2 The Revised Model – the OLG Model

The important finding from the overlapping generations model supports the permanent-income hypothesis of consumption stating that current consumption is geared towards expected lifetime income rather than towards current income. From the OLG model, we have derived that current aggregate consumption is a function of current lagged and expected future income, lagged price, expected future inflation rate, and lagged and expected money supply. In the meanwhile, the total demand for real balance is derived as a function of current and expected future income, expected future inflation rate, and expected future real balance. For the sake of simplification, the functions of money demand and consumption are described in their linear forms:

$$\begin{aligned}
(a)\quad m_t - p_t &= b_1 y_t - b_2 E_t y_{t+1} - b_3(E_t p_{t+1} - p_t) - b_4(E_t m_{t+1} - E_t p_{t+1}) \\
(b)\quad c_t &= c_1 y_{t-1} + c_2 y_t + c_3 E_t y_{t+1} + c_4 p_{t-1} + c_5(E_t p_{t+1} - p_t) \\
&\quad + c_6 E_t m_{t+1} - c_7 m_{t-1} + w_t \qquad (5.27)
\end{aligned}$$

where the coefficients b's and c's are positive and constant. We develop an alternative new model by replacing the money-demand and consumption functions of equations (2.2) and (2.7) in the T-B model with our new equations (5.27a) and (5.27b). In addition, lagged income is included in the aggregate supply equation (2.1). Our revised OLG model is developed and reduced to the following equations:

$$\begin{aligned}
&(a)\quad y_t = s_1 y_{t-1} + s_2(p_t - E_{t-1} p_t) + u_t \\
&(b)\quad m_t - p_t = b_1 y_t - b_2 E_t y_{t+1} - b_3(E_t p_{t+1} - p_t) - b_4(E_t m_{t+1} - E_t p_{t+1}) \\
&(c)\quad p_t = p_t^* + e_t \\
&(d)\quad \eta[(E_t e_{t+1} - E_{t-1} e_t) - (e_t - e_{t-1}) + (r_t^* - r_{t-1}^*) - (r_t - r_{t-1})] \\
&\qquad = -c_1 y_{t-1} + (1 - c_2) y_t - c_3 E_t y_{t+1} - c_4 p_{t-1} - c_5(E_t p_{t+1} - p_t) \\
&\qquad\quad - c_6 E_t m_{t+1} + c_7 m_{t-1} - w_t \qquad (5.28)
\end{aligned}$$

Like the revised CIA model described in the preceding section, the system is recursive. Equations (5.28a) and (5.28b) simultaneously determine the solution of income and price. Subsequently, the exchange rate is determined simply from equation (5.28c) once the solution of price is obtained. Finally, the interest rate is solved from equation (5.28d).

Any change in money supply has an immediate effect on income and price, through the money market. Given price (p_t), expected future price ($E_t p_{t+1}$), income ($E_t y_{t+1}$), and money supply ($E_t m_{t+1}$), the public holds a real balance greater than what they want. They will attempt to reduce the amount of money holdings by purchasing more goods. Therefore, the price level will rise and prices will deviate more than the firms have expected. Aggregate output will be induced to increase in response to the large price forecast error. The public may revise their expectations about future price, income, and money supply after introducing the new observations into the information set available to them. Price and income will adjust continuously, in order to restore the system to equilibrium.

5.2.1 The superior-information money-supply rule

As with the revised CIA model, there is no rational expectation solution of price in this model. Therefor there exists no optimal feedback money-supply rule when Murata's method is applied to the model. Following the assumptions mentioned in the preceding section, we will derive the optimal money-supply rule while positing that the monetary authority has information superior to that of the public. As described in the preceding section, the money-supply rule is designated as the general form

$$m_t = \overline{m} + a_1 u_t + a_2 p_t^* + a_3 w_t \tag{5.29}$$

Where $\overline{m}$ is the time trend of money supply. For convenience, set $\overline{m}$ equal to zero.

From equation (5.28a), we have the rational expectation solution of income that

$$\begin{aligned} E_{t-1}y_t &= s_1 y_{t-1} \\ E_t y_{t+1} &= s_1 y_t \\ E_{t-1}y_{t+1} &= s_1^2 y_{t-1} \end{aligned} \tag{5.30}$$

Taking rational expectations on both sides of equation (5.28b) and with (5.29) and (5.30), the rational expectation solution of price is

$$E_{t-1}p_t = \frac{s_1(b_1 - b_2 s_1)}{(1+b_3)} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-2-j} \tag{5.31}$$

Notice that $|\frac{1+b_3}{b_3-b_4}| < 1$ is required, in order to ensure the convergence of the sequence $\{y_{t-j}\}$ and the existence of the backward solution of $E_{t-1}p_t$. Equations (5.28a), (5.28b), and (5.29) can be reduced to

$$\begin{aligned} y_t = & \frac{1}{v}\{(1+b_3)s_1 y_{t-1} + s_2(a_1 u_t + a_2 p_t^* + a_3 w_t) \\ & + s_2(b_3 - b_4)[\frac{s_1(b_1 - b_2 s_1)}{1+b_3} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-1-j}] \\ & + s_2(1+b_3)[\frac{s_1(b_1 - b_2 s_1)}{1+b_3} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-2-j}]\} \end{aligned} \tag{5.32}$$

where $v \equiv 1 + b_3 + s_2(b_1 - b_2 s_1)$

Suppose that the government desires to minimize income variance, $\sum_{t=1}^{\infty} E_{t-1}(y_t - \overline{y})^2$, where $\overline{y}$ is a time trend or an arbitrage target value of y_t. In brief, the objective of the government is to minimize $E_{t-1}(y_t - \overline{y})^2$ by choosing the policy parameters a_1, a_2, and a_3. From equation (5.32),

$$a_1 = -\frac{(1+b_3)}{s_2}, \qquad a_2 = 0, \qquad a_3 = 0 \tag{5.33}$$

is easily obtained such that income variance is minimized. The superior-information money-supply rule is therefore written as

$$m_t = -\frac{(1+b_3)}{s_2}u_t \tag{5.34}$$

Equation (5.34) demonstrates that money supply must decrease if the economy undergoes an unanticipated increase in aggregate supply, or vice versa.

From equations (5.28a), (5.28b), (5.32), and (5.34), we have the reduced forms of y_t, p_t, and e_t:

$$\begin{aligned} (a) y_t &= s_1 y_{t-1} \\ (b) p_t &= -\frac{1}{s_2}u_t + \frac{s_1(b_1 - b_2 s_1)}{1+b_3}\sum_{j=0}^{\infty}\left(\frac{1+b_3}{b_3-b_4}\right)^{j+1} y_{t-2-j} \\ (c) e_t &= -\frac{1}{s_2}u_t + \frac{s_1(b_1 - b_2 s_1)}{1+b_3}\sum_{j=0}^{\infty}\left(\frac{1+b_3}{b_3-b_4}\right)^{j+1} y_{t-2-j} - p_t^* \end{aligned} \tag{5.35}$$

Under the optimal money-supply rule with superior information and rational expectations, the income process (5.35a) is simply a first-order autoregressive process and contains no random disturbance term. It means that income under this money-supply rule can be fully insulated from random shock.

The price and exchange rate processes (5.35a) and (5.35b) are autoregressive processes of income and contain a random disturbance term involving domestic supply shock. Of course, exchange rate contains a foreign price disturbance term, in addition to the domestic supply disturbance term. Even though price will be affected by all lagged income, the farther past income has the least effect on current price under the conditions of the weight $|\frac{b_1 - b_2 s_1}{1+b_3}| < 1$. Price will fall as unanticipated domestic supply increases. The downward pressure of price due to both unanticipated increases in aggregate supply and money supply leads to prices falling. Variations of price originating in domestic supply depend on the elasticity of aggregate supply with respect to price forecast error. Price will vary less if aggregate supply is more responsive to price forecast error.

Substituting from (5.31) and (5.35) into (5.28d), the reduced form of interest rate is

$$\begin{aligned} r_t &= r_{t-1} + [Y_{t-1}] + \frac{c_4}{\eta} p_{t-1} + (p_t^* - p_{t-1}^*) + (r_t^* - r_{t-1}^*) \\ &+ \frac{1}{s_2}(1 + \frac{c_5}{\eta}) u_t - \frac{1}{s_2}[1 - \frac{c_7(1+b_3)}{\eta}] u_{t-1} + \frac{w_t}{\eta} \end{aligned} \tag{5.36}$$

where

$$\begin{aligned} [Y_{t-1}] &\equiv [\frac{c_1 - (1-c_2)s_1 + c_3 s_1^2}{\eta} + (1 + \frac{c_5}{\eta}) \frac{s_1(b_1 - b_2 s_1)}{b_3 - b_4}] y_{t-1} \\ &+ [(1 + \frac{c_5}{\eta})(\frac{1+b_3}{b_3 - b_4}) - \frac{c_5}{\eta}](\frac{b_1 - b_2 s_1}{1 + b_3}) \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-2-j} \end{aligned}$$

The first-order interest rate process (5.36) is a very complicated autoregressive and moving average process (ARIMA) of income, price, and domestic supply disturbance, as well as all other random disturbances.

Suppose that the government is concerned only with price variation. In this case, the objective of the monetary authority is to minimize price variance, i.e., $\sum_{t=1}^{\infty} E_{t-1}(p_t - \bar{p})^2$ or $\sum_{t=1}^{\infty} E_{t-1}(p_t - E_{t-1}p_t)^2$, where $\bar{p}$ is an arbitrage targeted value or a time trend value. Equations (5.28a), (5.28b) and (5.29) can then be reduced to

$$\begin{aligned} p_t &= \frac{1}{v}\{a_1 u_t + a_2 p_t^* + a_3 w_t - (b_1 - b_2 s_1) s_1 y_{t-1} \\ &+ \frac{(b_3 - b_4) s_1 (b_1 - b_2 s_1)}{(1 + b_3)} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-1-j} \end{aligned}$$

$$+\frac{s_1 s_2(b_1 - b_2 s_1)^2}{1+b_3} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-2-j} - (b_1 - b_2 s_1) u_t \} \quad (5.37)$$

The policy parameters must be chosen as

$$a_1 = (b_1 - b_2 s_1) \ , \qquad a_2 = 0 \ , \qquad a_3 = 0 \quad (5.38)$$

such that price variance is minimized. The optimal superior-information money supply rule with the minimum-price-variance criterion is written as

$$m_t = (b_1 - b_2 s_1) u_t \quad (5.39)$$

The optimal superior-information money-supply rule (5.39) indicates that changes in money supply respond to current random domestic supply disturbances. It provides the policy implication that the monetary authority must decrease money supply in order to eliminate the impact of the increase in domestic supply disturbance on price, assuming that the economy has strong feedback from lagged output to current output and can overcome the relative responsiveness of money demand to current and expected income ($s_1 > b_1/b_2$). Consequently, the upward pressure of price due to the unanticipated increase in output is intended to be offset by an induced lowering of price through a decrease in money supply.

Substituting (5.39) into (5.28a), (5.28b), and (5.28c), the reduced forms of income, price, and exchange rate are obtained:

$$\begin{aligned}
(a) y_t &= s_1 y_{t-1} + u_t \\
(b) p_t &= \frac{s_1(b_1 - b_2 s_1)}{1+b_3} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-2-j} \\
(c) e_t &= \frac{s_1(b_1 - b_2 s_1)}{1+b_3} \sum_{j=0}^{\infty} (\frac{1+b_3}{b_3 - b_4})^{j+1} y_{t-2-j} - p_t^* \qquad (5.40)
\end{aligned}$$

Income process (5.40a) is simply a first-order autoregressive process with an additive constant variance domestic supply error term. The processes for price and exchange rate, (5.40b) and (5.40c), are processes of all lagged income with decreasing weights. It is obvious that the price and exchange rate can be fully insulated from the domestic supply disturbance under the superior-information money rule with the minimum-price-variance criterion.

The reduced form of the interest rate is easily obtained from (5.40) and (5.28d):

$$\begin{aligned} r_t &= r_{t-1} + (Y_{t-1}) + (1 + \frac{c_4}{\eta})p_{t-1} - (\frac{1 - c_1 - s_1 c_3}{\eta})u_t \\ &\quad - \frac{c_7}{u}(b_1 - b_2 s_1)u_{t-1} + (p_t^* - p_{t-1}^*) + (r_t^* - r_{t-1}^*) + \frac{w_t}{\eta} \end{aligned} \tag{5.41}$$

where (Y_{t-1}) is the weighted sum of all lagged income. Like the superior-information money-supply rule with the minimum income variance, the first-order process for interest rate under the rule with the minimum price variance criterion is a very complicate autoregressive and moving average process, which takes into account lagged income, price, and all random disturbances of domestic and foreign origin.

5.3 The Fixed Money Rule

Assume that the monetary authority adopts the fixed money-supply rule,

$$m_t = g \qquad \forall t$$

From equations (5.28a), (5.28b) and from $m_t = g$, the rational expectation solution of p_t, E_{t-1}, p_t is obtained such that

$$E_{t-1}p_t = \frac{s_1(b_1 - b_2 s_1)}{1 + b_3} \sum_{j=0}^{\infty} (\frac{1 + b_3}{b_3 - b_4})^{j+1} y_{t-2-j} - (\frac{b_3 - b_4}{1 + b_3})g \tag{5.42}$$

Substituting (5.42) into (5.28a), (5.28b), and (5.28c), we obtain the reduced forms of income, price, and exchange rate:

$$\begin{aligned}
(a) y_t &= \frac{s_2}{v}\frac{(1+b_4)(1+2b_3-b_4)}{(1+b_3)} g + s_1 y_{t-1} + \frac{1+b_3}{v} u_t \\
(b) p_t &= c + \frac{s_1(b_1-b_2s_1)}{1+b_3}\sum_{j=0}^{\infty}\left(\frac{1+b_3}{b_3-b_4}\right)^{j+1} y_{t-2-j} - \frac{(b_1-b_2s_1)}{v} u_t \qquad and \\
(c) e_t &= c + \frac{s_1(b_1-b_2s_1)}{1+b_3}\sum_{j=0}^{\infty}\left(\frac{1+b_3}{b_3-b_4}\right)^{j+1} y_{t-2-j} - \frac{(b_1-b_2s_1)}{v} u_t \\
&\quad - p_t^* \qquad (5.43)
\end{aligned}$$

where c is a constant term and $v \equiv 1 + b_3 + s_2(b_1 - b_2 s_1)$.

Like the superior-information rule with the minimum-price-variance criterion, income is a first-order autoregressive process and contains a random error term involving domestic supply shock. In addition, price and exchange rate are autoregressive processes of income. Unlike the superior-information rules in the preceding section, domestic supply shock turns out to effect all endogenous variables. More specifically, under the fixed money supply, random domestic supply shock impacts all endogenous variables. No endogenous variables can be insulated from domestic supply shock.

The reduced form of r_t is

$$\begin{aligned}
r_t &= Z + r_{t-1} + \{Y_{t-1}\} + \frac{c_4}{\eta} p_{t-1} + \left(1+\frac{c_5}{\eta}\right)\left(\frac{b_1-b_2s_1}{v}\right) u_t \\
&\quad -\left(1+\frac{c_4}{\eta}\right)\left(\frac{b_1-b_2s_1}{v}\right) u_{t-1} + (r_t^* - r_{t-1}^*) + (p_t^* - p_{t-1}^*) + \frac{w_t}{\eta} \qquad (5.44)
\end{aligned}$$

where Z is a constant term. $\{Y_{t-1}\}$ is a weighted sum of all lagged income. As with the preceding money rule, the interest rate process is a complicated autoregressive and moving average process. All random disturbances impact interest rate

Table 5.2: The responses of endogenous variables to domestic supply shock (u_t), under the alternative money supply rules

	minimun-income-variance criterion	minimun-price-variance criterion	the fixed rule
y_t	0	1	$\frac{1+b_3}{v}$
p_t	$-\frac{1}{s_2}$	0	$-\frac{(b_1-b_2s_1)}{v}$
e_t	$-\frac{1}{s_2}$	0	$-\frac{(b_1-b_2s_1)}{v}$
r_t	$-\frac{1}{s_2}(1+\frac{c_5}{\eta})$	$-\frac{(1-c_1-s_1c_3)}{\eta}$	$-(1+\frac{c_5}{\eta})(\frac{b_1-b_2s_1}{v})$ $v \equiv 1+b_3+s_2(b_1-b_2s_1)$

regardless of the origin of the random disturbance. In this economy, interest rate appears more erratic than other endogenous variables.

The relative performances of these three money supply rules can be compared according to the response of y_t to random shocks. The responses of all endogenous variables to domestic supply disturbance (u_t) are described in Table 5.2.

As shown in Table 5.2, price and exchange rate under the rule with minimum price variance is fully insulated from domestic supply shock, in contrast with both the minimum-income-variance rule and the fixed rule. Moreover, income is completely insulated from u_t. For an economy with high elasticity of aggregate output with respect to price forecast error, it appears better to adopt the minimum-income-variance criterion rule. On the contrary, for an economy with low elasticity of aggregate output with respect to price forecast error, it appears better to adopt the rule with the minimum-price-variance criterion.

In this revised OLG macroeconomic model, where the government has information superior to that of the public, the optimal money-supply rule is chosen in accordance with current domestic supply shock. Superior-information rules are dominant over fixed money rules in the sense that the economy can be insulated from random shock.

In this context, stabilization policy does not mean the elimination of the business cycle (or serial correlation in income). Remember that there exists no optimal feedback rule in this model. Rather, 'optimal' is used in the sense of insulation from random shocks.

The probability distributions of income, even of other endogenous variables, are affected by the stabilization policy of the superior-information rule. It indicates that monetary stabilization policy under the superior-information rule is more effective than the fixed rule in stabilizing the economy.

6 CONCLUSION

The main purpose of this study is the investigation of the effectiveness of monetary policy in changing probability distributions of the endogenous variables for an economy with rational expectations. Our emphasis is the discussion of an 'optimal' monetary policy in which fluctuations of the economic system are minimized over time.

The asymptotic variance of the output, price, and exchange rates are calculated for four cases of money supply policies in the T-B model: (1) where the money supply follows a random walk; (2) where the derived deterministic feedback money supply rule incorporates the public's expectations into the formulation of a control rule for money supply and includes three targeted variables (output, price, and exchange rate) in the social welfare loss function; (3) where the derived deterministic feedback money supply control rule includes one target variable (output) in the social welfare loss function; and (4) where the fixed money supply rule holds money constant over time.

The responses of all the endogenous variables to a variety of random disturbances can be easily captured by the parameter coefficients associated with corresponding random disturbance terms in the final form of the endogenous variables.

The dynamic optimal control method has been applied to different stochastic macroeconomic models. The salient results under alternative monetary policies and models are:

(1) In the T-B model, the case of a one-target-variable (y_t) with optimal feedback control setting is superior to the case of three target variables, in regards to stabilizing output.

(2) In the T-B model, the income, price, and interest rates are exogenous processes under the feedback money supply rule, but price and interest rate become time-dependent convergent processes rather than exogenous processes. This indicates that the feedback money supply rule is superior to the fixed rule.

(3) In regards to the T-B model, feedback monetary policy is not ineffective in an economy with rational expectations. It may affect the probability distribution of output, price, and other endogenous variables.

(4) Murata's method fails to apply to our revised models, which have been developed according to important insights in the CIA and OLG models. In these two revised models, there is no optimal feedback, as it is assumed that the public and the monetary authority share the same information in forming their expectations. It yields the Sargent-Wallace result, which substantiates Friedman's contention that the monetary authority should abandon attempts to pursue an activist stabilization policy.

(5) Under the assumption that the monetary authority has information superior to that of the public, we have derived an alternative money supply rule for our revised model. The establishment of a superior-information money rule suggests that change in money supply is in accordance with current random shocks. Either income or price can be completely insulated from the domestic supply shock.

The probability distribution of income, price, and other endogenous variables are affected by our derived superior-information money rules. We show that the superior information money supply rule is more powerful than the fixed money rule in stabilizing the economy.

(6) The structure of the economy does really matter in the choice of an optimal money supply rule. What the optimal money supply rule should depend on the structure of the economy. The optimal money supply rule derived for one economy is not necessarily the optimal money supply rule for economy with a different structure.

The variance of the output differs with alternative money supply rules. The analysis of the feedback money supply in the T-B model demonstrates that our derived optimal control rule of money supply, which incorporates public expectations, has the effect on variation of output that arises from random disturbances of domestic and foreign origin. The probability distribution of output and other endogenous variables are influenced by the government policy rule. This result is contrasted with the policy ineffectiveness proposition. The deterministic feedback money supply appears effective in the T-B model with rational expectations, but ineffective in our revised models, which supports the ineffectiveness proposition. On the other hand, the superior-information money rule appears to be effective and clearly dominates the fixed rule in our revised models. It can be stated conclusively that relative performances of alternative money supply rules depend on the structure of the economy and the formation of rational expectations. The structure of the economy does matter when the optimal money rule is being determined.

It is questionable whether output variability is an appropriate stabilization criterion. The alternative criterion of minimization of the supplier's expectational error has been considered in the original model. The derived optimal money supply rule is formulated by minimizing the sum of the squares of the price forecast error, which leads to the attainment of the employment with complete information.

The magnitudes of the response of all endogenous variable to all random shocks could be examined by tracing the time paths of the endogenous variables from the numerical solution in the simulation experiments. This would provide a clear policy implication. But the numerical solutions under the alternative money supply policies have not yet been arrived at. This will be the subject of future research.

7 BIBLIOGRAPHY

Barro, R. J. "A Stochastic Equilibrium Model of an Open Economy under Flexible Exchange Rates." *Quarterly Journal of Economics* 92, No. 1 (February 1978): 149–164.

Bilson, J. F. O. "Rational Expectations and the Exchange Rate." *In The Economics of Exchange Rates*, ed. J. A. Frenkel and H. G. Johnson. Reading, Mass.: Addison-Wesley, 1978.

Black, S. W. "The Effect of Alternative Intervention Policies on Variability of Exchange Rates : The Harrod Effect." *In Exchange Rate Management Under Uncertainty*, ed. J. S. Bhandari. Cambridge, Mass.: MIT Press, 1985.

Brock, W. A. and Scheinkman, J. A. "Some Remarks on Monetary Policy in an Overlapping Generations Model." *In Models of Monetary Economics,* ed. J. H. Kareken and N. Wallace, pp. 211–232. Minneapolis, Minnesota: Federal Reserve Bank of Minneapolis , 1980.

Clower, R. W. "A Reconsideration of the Microfoundations of Monetary Theory." *Western Economic Journal* 6 (December 1967): 1–8.

Dornbusch, R. "Expectations and Exchange Rate Dynamics." *Journal of Political Economy* 84 (December 1976): 1161–1176.

Driskill, R. and McCafferty, S. A. "Exchange Rate Variability, Real and Monetary Shocks, and the Degree of Capital Mobility under Rational Expectations." *Quarterly Journal of Economics* 95 (November 1980): 577–586.

Driskill, R. and McCafferty, S. A. "Exchange Market Intervention under Rational Expectations with Imperfect Capital Substitutability." *In Exchange Rate Management under Uncertainty*, ed. J. S. Bhandari. Cambridge, Mass.: MIT Press, 1985.

Friedman, M. *A Program for Monetary Stability*. New York: Fordham University Press, 1959.

Friedman, M. " A Monetary and Fiscal Framework for Economic Stability." *American Economic Review* 38 (June 1948): 245–264.

Friedman, M. "The Role of Monetary Policy." *American Economics Review* 58 (March, 1968): 1–17.

Kareken, J. H. and Wallace, N. eds. *Models of Monetary Economics*. Minneapolis, Minnesota: Federal Reserve Bank of Minneapolis, 1980.

Kareken, J. H. and Wallace, N. "Samuelson's Consumption-Loan Model with Country-Specific Fiat Monies. *Research Department Staff Report* 24. Minneapolis, Minnesota: Federal Reserve Bank of Minneapolis, 1978.

Kareken, J. H. and Wallace, N. "On the Indeterminancy of Equilibrium Exchange Rates." *Quarterly Journal of Economics* 96 (1981): 207–222.

Kyland, F. E. and Prescott, E. C. "Rules Rather than Discretion : The Inconsistency of Optimal Plans." *Journal of Political Economy* 85 (1977): 473–493.

Levine, P. and Currie, D. "The Design and Feedback Rules in Linear Stochastic Rational Expectations Models." *Journal of Economic Dynamics and Control* 11 (1987): 1–28.

Levine, P. and Currie, D. "Optimal Feedback Rules in an Open Economy-macro Model with Rational Expectations." *European Economic Review* 27 (1985): 141–163.

Lucas, R. E., Jr. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4 (April, 1972): 103–124.

Lucas, R. E., Jr. "Some International Evidence on Output-Inflation Trade-offs." *American Economic Review* 63 (June 1973): 326-334.

Lucas, R. E., Jr. "Econometric Policy Evaluation : A Critique." Journal of Monetary Economics 2, Supplement (1976): 19–46.

Lucas, R. E., Jr. "Interest Rates and Currency Prices in a Two-Country World."

Journal of Monetary Economics 10 (1982): 335–359.

McCallum, B. T. "Rational Expectations and Macroeconomic Stabilization Policy." Journal of Money, Credit and Banking 12 (November 1980): 716–746.

McCallum, B. T. and Whitaker, J. K. "The Effectiveness of Fiscal Feedback Rules and Automatic Stabilizers under Rational Expectations." *Journal of Monetary Economics* 5 (1979): 171–186.

Murata, Y. *Optimal Control Methods for Linear Discrete-Time Economic Systems.* New York: Springer–Verlag, 1982.

Phelps, E. S. "Phillips Curves, Expectations of Inflation, and Optimal Unemployment Over time." *Economica* 34 (August 1967): 254–281.

Poole, W. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model." *Quarterly Journal of Economics* 84 (May 1970): 197–216.

Samuelson, P. A. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money." *Journal of Political Economy* 66 (1958): 467–482.

Sargent, T. J. "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment." *Brookings Papers on Economic Activity* 2 (1973): 429–472.

Sargent, T. J. "A Classical Macroeconometric Model for the United States." *Journal of Political Economy* 84 (April 1976): 207–237.

Sargent, T. J. *Macroeconomic Theory.* New York: Academic Press, 1979.

Sargent, T. J. *Dynamic Macroeconomic Theory.* Cambridge, Mass.: Harvard University Press, 1987.

Sargent, T. J. and Wallace N. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *Journal of Political Economy* 83 (April 1975): 241–254.

Stockman, A. C. "A Theory of Exchange Rate Determination." *Journal of Political Economy* 83 (1980): 673–698.

Stockman, A. C. and Svensson, L. E. O. "Capital Flows, Investment, and Exchange Rates." *Journal of Monetary Economics* 19 (1987): 171–202.

Svensson, L. E. O. "Money and Asset Prices in a Cash-in-Advance Economy." *Journal of Political Economy* 93 (1985a): 919–944.

Svensson, L. E. O. "Currency Prices, Terms of Trade, and Interest Rates." *Journal of International Economics* 18 (1985b): 17–41.

Tsiang, S. C. "Liquidity Preference and Loanable Funds Theory, Multiplier and Velocity Analysis: A Synthesis" *American Economic Review* 46 (1956): 540–564.

Turnovsky, S. J. and Bhandari, J. S. "The Degree of Capital Mobility and the Stability of an Open Economy under Rational Expectations." *Journal of Money, Credit and Banking* 14 (August 1982): 303–326.

Wallace, N. "The Overlapping-Generation Model of Fiat Money." In *Models of Monetary Economics,* ed. J. H. Kareken and N. Wallace, pp. 49–82. Minneapolis, Minnesota: Federal Reserve Bank of Minneapolis, 1980.

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